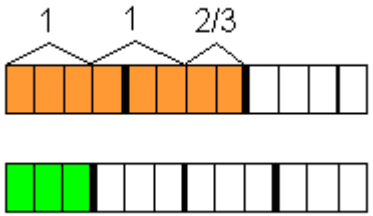
Mathematics Standards

GRADE: 6

Big Idea 1: BIG IDEA 1 Develop an understanding of and fluency with multiplication and division of fractions and decimals.	
BENCHMARK CODE	**BENCHMARK**
MA.6.A.1.1	Explain and justify procedures for multiplying and dividing fractions and decimals. *Remarks/Examples*: For division of fractions, students might use drawings, manipulatives, and symbolic notation to describe how and explain why they can find a common denominator and then divide just the numerators to find the quotient. Example: In order to divide 2/3 by 1/4 , a student may reason that 2/3 = 8/12 and 1/4 = 3/12. So, (2/3)÷(1/4) is equivalent to (8/12)÷(3/12), which gives the same result as 8÷3=2 2/3. The following picture is a representation that matches the above explanation:  In the following fraction multiplication examples, students may use drawings or physical objects to represent the problems and explain their solution. Example 1: One-half of your yard is garden. One- fourth of your garden is a vegetable garden. What fraction of your yard is a vegetable garden? Draw a picture and write a number sentence that both describe the problem and solution.

	Pizza Parlor Scenarios Example 2: A cook made four pizzas that had 3/5 of a package of mushrooms on each. How many packages of mushrooms were used? Example 3: Sue ate some pizza. 2/3 of a pizza is left over. Jim ate 3/4 of the left over pizza. How much of a whole pizza did Jim eat? Example 4: A party dessert pizza measures 2/3 of a yard by 3/4 of a yard. How much of a square yard is the party dessert pizza? Example 5: There was 4/5 of a pound of pizza dough leftover in the freezer from the previous day. The cook thawed out 3/8 of the leftover dough. How much of a pound of dough did the cook thaw? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.1.2	Multiply and divide fractions and decimals efficiently. *Remarks/Examples*: Students may learn techniques such as mental math or specified algorithms to perform these operations. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.6.A.1.3	Solve real-world problems involving multiplication and division of fractions and decimals. *Remarks/Examples*: This standard includes finding the solution to multi-step problems. Example: How many quarter-pound hamburgers can be made from 3 1/2 pounds of ground beef? *Cognitive Complexity/Depth of Knowledge Rating:* High

Big Idea 2: BIG IDEA 2

Connect ratio and rates to multiplication and division.

BENCHMARK CODE	BENCHMARK
MA.6.A.2.1	Use reasoning about multiplication and division to solve ratio and rate problems.

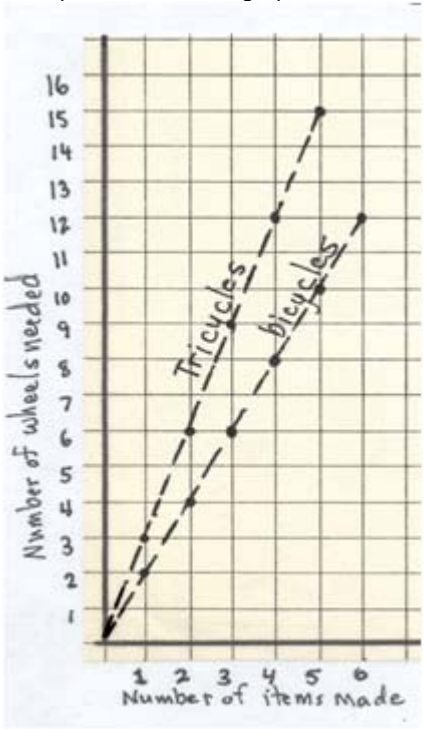
	Remarks/Examples: Example: Four items cost $5.00 and all items are the same price. Explain how to find the cost for 9 items. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.6.A.2.2	Interpret and compare ratios and rates. *Remarks/Examples*: Example: Jessica made 8 out of 24 free throws. Bob made 5 out of 20 free throws. Who has the highest free throw ratio? Ratios may be represented in various forms such as simple drawings or multiplication tables. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Big Idea 3: BIG IDEA 3

Write, interpret, and use mathematical expressions and equations.

BENCHMARK CODE	BENCHMARK
MA.6.A.3.1	Write and evaluate mathematical expressions that correspond to given situations. *Remarks/Examples*: Example: A plant is 3 cm high on Day 1. Each day after that the plant grows 2 cm taller. Assume that the plant grows at the same rate. Make a table and graph that show the height of the plant for Days 1 through 10. Write an expression to show the height on day n. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.3.2	Write, solve, and graph one- and two- step linear equations and inequalities. *Remarks/Examples*: The context should include patterns, models and relationships. Students should explore how "greater than or equal to" and strictly "greater than" are similar and different. A number line is a useful tool for modeling situations and inequalities such as "You have to be at least 40 inches tall to a ride roller coaster." and "x = 40". Graphing on coordinate plane is still limited to the first quadrant, but they can explore negative and positive integers on number line. Example: The height of a tree was 7 inches in the year 2000. Each year the same tree grew an additional 10 inches. Write an equation to show the height *h* of the tree in *y* years. Let y be the number of years after the year 2000. Graph the height of the tree for the first 20 years.

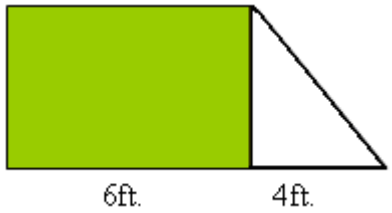
	The most literal equation might be 7 + 10y = h. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.3.3	Work backward with two-step function rules to undo expressions. *Remarks/Examples*: Example: Sam set a function machine to multiply by 3, and then to add 4. He showed his chart to Wanda. How can Wanda find the missing input number? IN OUT 1 7 5 19 20 64 100 304 ? 79 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.3.4	Solve problems given a formula. *Remarks/Examples*: Example: The pressure exerted by a solid object on a solid surface can be calculated by using the formula, , where the variables P, F, and A represent pressure, force, and area respectively. A newly refinished wood floor can withstand a pressure of up to 40 pounds per square inch without sustaining damage. A 120 pound woman with high heels and a 240 pound man with flat heels each enter this room. Assume that at some point all of their weight is supported equally by the heels of both of their shoes. Given that each of the woman's heels occupies an area of 0.25 in^2 and each of the man's heels occupies an area of 12 in^2, find out each person's potential for causing damage to the wood floor. Justify your answer. If a 15,000 pound African elephant with feet that each has an area of 100 in^2 were to stand on this floor, would it cause damage to the floor? Explain your answer. Compare the three cases with each other. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.3.5	Apply the Commutative, Associative, and Distributive Properties to show that two expressions are equivalent. *Remarks/Examples*: Example: Is 7(m+2) the same as 7m + 2 or 7m + 14? Explain your choice. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

MA.6.A.3.6	Construct and analyze tables, graphs, and equations to describe linear functions and other simple relations using both common language and algebraic notation. *Remarks/Examples*: Example 1: Each unicycle made needs 1 wheel. Explain why *w=u* where *w* is the number of wheels and *u* is the number of unicycles describes this relationship. Example 2: Each bicycle made needs 2 wheels. Explain why *w=2b* where *w* is the number of wheels and *b* is the number of bicycles describes this relationship. Example 3: Each tricycle made needs 3 wheels. Explain why w=3t where *w* is the number of wheels and *t* is the number of tricycles describes this relationship. Example 4: Below is a graph of the relationships in Examples 2 and 3. Explain why one of the lines is steeper than the other line.  What would the graph of w=u look like? *Cognitive Complexity/Depth of Knowledge Rating:* High

Supporting Idea 4: Geometry and Measurement

Geometry and Measurement

BENCHMARK CODE	BENCHMARK

MA.6.G.4.1	Understand the concept of Pi, know common estimates of Pi (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles. *Remarks/Examples*: Using various circular objects, students determine that the ratio of circumference to diameter approximates the value of Pi. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.G.4.2	Find the perimeters and areas of composite two-dimensional figures, including non-rectangular figures (such as semicircles) using various strategies. *Remarks/Examples*: Example: Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into shapes that approximates a parallelogram. Example: Students might trace their foot on a piece of grid paper and use the full squares and the partial squares to estimate the area of the bottom of their foot. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.G.4.3	Determine a missing dimension of a plane figure or prism given its area or volume and some of the dimensions, or determine the area or volume given the dimensions. *Remarks/Examples*: Example: The volume of a rectangular prism is 112 cubic cm. The length is 7 cm, and the height is 8 cm. What is the depth of the prism? Example: The figure below shows the floor of a living room. The rectangular part is covered with a carpet that covers a 22 square feet area. The house owner wants to cover the triangular part with carpet as well. Use the information provided in figure to determine the minimum additional carpet that will need to be purchased to cover the floor. 

	Cognitive Complexity/Depth of Knowledge Rating: Moderate

Supporting Idea 5: Number and Operations

Number and Operations

BENCHMARK CODE	BENCHMARK
MA.6.A.5.1	Use equivalent forms of fractions, decimals, and percents to solve problems. *Remarks/Examples*: Example: John scored 75% on a test and Mary has 8 out of 12 correct on the same test. Each test item is worth the same amount of points. Who has the better score? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.5.2	Compare and order fractions, decimals, and percents, including finding their approximate location on a number line. *Remarks/Examples*: Example: Approximate the location of each of these values on a number line: 2/3, 0.57, and 0.575. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.6.A.5.3	Estimate the results of computations with fractions, decimals, and percents, and judge the reasonableness of the results. *Remarks/Examples*: Example: Amy bought 5 notebooks at $3.61 each. She estimated how much she needs to pay and gave the cashier $15. Is Amy's estimation appropriate? Explain your reasoning. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Supporting Idea 6: Data Analysis

Data Analysis

BENCHMARK CODE	BENCHMARK
MA.6.S.6.1	Determine the measures of central tendency (mean, median, mode) and variability (range) for a given set of data. *Remarks/Examples*: Students should make frequency tables for numerical or categorical data, grouping data in different ways to investigate how different groupings describe the data. This is the first time in 2007 Florida mathematics standards that students are expected to use mean, median, mode, and range in a formal sense to describe a set of data. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.6.S.6.2	Select and analyze the measures of central tendency or variability to represent, describe, analyze, and/or summarize a data set for the purposes of answering questions appropriately.

	Remarks/Examples: A teacher can give students data sets that contain test/quiz grades for hypothetical classes. Students are asked to calculate and compare the class mean, median, mode, and range and discuss the effects of any outliers on the measures of central tendency. *Cognitive Complexity/Depth of Knowledge Rating:* High

GRADE: 7

Big Idea 1: BIG IDEA 1

Develop an understanding of and apply proportionality, including similarity.

BENCHMARK CODE	BENCHMARK
MA.7.A.1.1	Distinguish between situations that are proportional or not proportional, and use proportions to solve problems. *Remarks/Examples*: Example 1: Two snakes, Moe and Joe, are each measured at two points in time. The first time, Moe is 3 inches long and Joe is 4 inches long. One year later, Moe is 5 inches long and Joe is 6 inches long. Which snake grew more? Maria believes that both snakes grew the same amount. Tom believes that Moe grew more. Explain under what circumstances either explanation could be correct. (In absolute terms they grew the same amount, which is not a proportional relationship; in relative terms one grew more than the other, which is a proportional relationship.) Example 2: A recipe calls for 3 cups of flour and 2 eggs. If you wanted to increase the recipe and use 9 cups of flour, how many eggs would you need to use to keep the same ratio of flour to eggs? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.7.A.1.2	Solve percent problems, including problems involving discounts, simple interest, taxes, tips, and percents of increase or decrease. *Remarks/Examples*: Example: A merchant buys CDs for $11 wholesale and marks up the price by 35%. What is the retail price? Example: You are at a party with 100 people. 99% of the people are FSU fans. Some of the FSU fans left the party and now 98% of the people are FSU fans. How many people are still at the party? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.7.A.1.3	Solve problems involving similar figures. *Remarks/Examples*: Example: Rectangle *A* and rectangle *B* are similar. The lengths of congruent sides of rectangles *A* and *B* are 6 in. and 5 in., respectively. If the perimeter of rectangle *A* is 30 in., then what is the perimeter of rectangle *B*?

	Cognitive Complexity/Depth of Knowledge Rating: High
MA.7.A.1.4	Graph proportional relationships and identify the unit rate as the slope of the related linear function. *Remarks/Examples*: In a linear relation, the vertical change (change in *y*-value) per unit of horizontal change (change in *x*-value) is always the same and this ratio ("rise over run") is called the slope of the function. Example: A babysitter earns $5 per hour. Draw a graph of money earned versus time. Find the numerical value of the slope and interpret it in words. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.A.1.5	Distinguish direct variation from other relationships, including inverse variation. *Remarks/Examples*: Direct variation between y and x is when y/x=k where k is a constant, or equivalently y=kx. Indirect variation is when xy=k where k is a constant, or equivalently y=k/x. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.A.1.6	Apply proportionality to measurement in multiple contexts, including scale drawings and constant speed. *Remarks/Examples*: The student might convert among different units of measurement to solve problems involving rates. Example 1: On a floor plan of your school, your classroom is 9 inches long and 6 inches wide. If the scale is 1 inch = 3 ft., what is the width of your classroom in feet? Explain your answer. Example 2: You have a 4 in. by 5 in. photograph and you want to enlarge it to an 8 in. by 10 in. photograph. Roberto thinks that the new picture is four times as big as the old one. Dora thinks that the new picture is twice as big as the old one. Explain their thinking. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Big Idea 2: BIG IDEA 2

Develop an understanding of and use formulas to determine surface areas and volumes of three-dimensional shapes.

BENCHMARK CODE	BENCHMARK
MA.7.G.2.1	Justify and apply formulas for surface area and volume of pyramids, prisms, cylinders, and cones.

	Remarks/Examples: Students should be limited to prisms, pyramids and cylinders when calculating surface area, and prisms, pyramids, cylinders and cones when calculating volume. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.G.2.2	Use formulas to find surface areas and volume of three-dimensional composite shapes. *Remarks/Examples*: This extends the work of grade 5 to using general formulas to compute the solutions for a variety of shapes. The figure being composed or decomposed may include circles or parts of circles. Example: Given a 3-Dimensional "E" shaped figure with labeled side lengths, find the surface area of the figure. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Big Idea 3: BIG IDEA 3

Develop an understanding of operations on all rational numbers and solving linear equations.

BENCHMARK CODE	BENCHMARK
MA.7.A.3.1	Use and justify the rules for adding, subtracting, multiplying, dividing, and finding the absolute value of integers. *Remarks/Examples*: Remarks: Problems should be solved using concrete or pictorial representations of models, tables, and graphs, instead of using algebraic symbolism. Example: Use the information provided in the table below to respond to each question. Date Description Amt WD Amt Dep Bal 1/1/08 Beg Bal $500.00 1/5/08 Ret Ck Fee $25 475.00 1/5/08 Ret Ck Fee $25 450.00 1/6/08 Resolution $25 475.00 1/6/08 Resolution $25 500.00 Mary was charged $25 each for 2 checks that bounced. Explain why the expression 2 x (-25) = -150 describes the situation.

	Mary had the problem of the bounced checks resolved and didn't have to pay the penalty of $25. She wrote (-2) x (-25) = 50 to fix her checkbook. Explain why a negative number multiplied by a negative number gives a positive number in this situation. Example: Use a set of integer chips to model one method for evaluating -5 + 8 – (-2). Explain your work. Possible Answer: I began with a group of 5 negative integer chips, and then combined it with another group (added) of 8 positive integer chips, and then I removed (subtracted) two negative integer chips. Finally, I removed three "zero pairs" (one positive and one negative integer chips), since -1 + 1 = 0. My final answer was positive 5. - - - - - + + + + + + + + - - - + + + + + + + + + + + + + *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.A.3.2	Add, subtract, multiply, and divide integers, fractions, and terminating decimals, and perform exponential operations with rational bases and whole number exponents including solving problems in everyday contexts. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.A.3.3	Formulate and use different strategies to solve one-step and two-step linear equations, including equations with rational coefficients. *Remarks/Examples*: Example: It costs an initial fixed cost of $2 plus an additional $1.50 per mile to rent a taxi. Which equation represents the method for calculating the total cost of a taxi ride? What is the total cost for a 5-mile trip? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.A.3.4	Use the properties of equality to represent an equation in a different way and to show that two equations are equivalent in a given context. *Remarks/Examples*:

	Properties of equality explain the following results: · A balanced equation will remain balanced if you add, subtract, multiply or divide (excluding division by zero) both sides by the same number. · A quantity equivalent to another quantity can be substituted for it. Example 1: What is another way to express the following equation? 3x + 14 = x + 30 Example 2: Why is 2x + 4 = x + 6 the same as 2x = x + 2 ? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Supporting Idea 4: Geometry and Measurement

Geometry and Measurement

BENCHMARK CODE	BENCHMARK
MA.7.G.4.1	Determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures, and apply these relationships to solve problems. *Remarks/Examples*: See Example 2 in benchmark MA.7.A.1.6. The linear scale factor is 2. The areas of the two figures are related by a factor of 4 (2 squared). If this pattern was continued for a 3-dimensional figure, the volumes would be related by a factor of 8 (2 cubed). Students should encounter this concept in different contexts, and they should be encouraged to recognize the patterns themselves rather than be told about the relationship first. Example: You have two circles with circumference pi and 4pi. What is the ratio of the areas of the circles? What is the ratio of the diameters? What is the ratio of the radii? *Cognitive Complexity/Depth of Knowledge Rating:* High

MA.7.G.4.2	Predict the results of transformations, and draw transformed figures with and without the coordinate plane. *Remarks/Examples*: Students should recognize that reflections, transformations, and rotations result in congruent figures. Other transformations (such as dilations) may not preserve congruency. Example 1: Draw the triangle with vertices (0,0), (3,0), (0,4). Translate (slide) the triangle 2 units to the right. What are the coordinates of the vertices of the new triangle? Example 2: What happens to a figure drawn on a coordinate plane if each of its vertices' coordinates is multiplied by 2? What if they are multiplied by 1/4th? What about -2? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.G.4.3	Identify and plot ordered pairs in all four quadrants of the coordinate plane. *Remarks/Examples*: Quadrants 2, 3, and 4 are introduced for the first time in 7th grade. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.7.G.4.4	Compare, contrast, and convert units of measure between different measurement systems (US customary or metric (SI)), dimensions, and derived units to solve problems. *Remarks/Examples*: Example 1: You ride your bike from your house to the beach and home again. At the end of your trip, your bicycle odometer reads 8km. How many miles did you ride? Example 2: How many cm3 are in a 2-liter bottle of soda? *Cognitive Complexity/Depth of Knowledge Rating:* High

Supporting Idea 5: Number and Operations

Number and Operations

BENCHMARK CODE	BENCHMARK
MA.7.A.5.1	Express rational numbers as terminating or repeating decimals. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.7.A.5.2	Solve non-routine problems by working backwards. *Remarks/Examples*: Solving non-routine problems involves creativity and critical thinking. Solution methods for non-routine problems are not prescribed.

	They may involve multiple representations, and are challenging for the learner. Example: Alex had some marbles. On his birthday, his father doubled the number of his marbles. Alex gave 5 marbles to his best friend. Then he divided the remaining marbles into three equal groups and shared them with his two brothers. Each brother got 11 marbles. What was the original number of marbles that Alex had before his birthday? Did he make a good choice of sharing his marbles? What strategy would you use if you were Alex? *Cognitive Complexity/Depth of Knowledge Rating:* High

Supporting Idea 6: Data Analysis

Data Analysis

BENCHMARK CODE	BENCHMARK
MA.7.S.6.1	Evaluate the reasonableness of a sample to determine the appropriateness of generalizations made about the population. *Remarks/Examples*: Example: You asked 10 of your classmates what is their favorite university in Florida. Five of them said Florida International University. Based on your sample, can we assume that FIU is the favorite university of approximately half of the students in your school? In your class? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.7.S.6.2	Construct and analyze histograms, stem-and-leaf plots, and circle graphs. *Remarks/Examples*: Students can represent the same data with different types of graphs and discuss the appropriateness of each graph based on the source of the data and the information required. An example of a stem-and-leaf plot for the data set (34, 30, 38, 42, 67, 68, 68, 56, 54, 34, 82, and 85) is as follows: Legend: 3| 234 means scores of 32, 33, and 34 **3 | 0,4,4,8** **4 | 2** **5 | 4,6** **6 | 7,8,8**

	7 **8**	 **2,5**

What is the median of the data set? What is the mode of the data set?

Cognitive Complexity/Depth of Knowledge Rating: Moderate

Supporting Idea 7: Probability **Probability**	
BENCHMARK CODE	**BENCHMARK**
MA.7.P.7.1	Determine the outcome of an experiment and predict which events are likely or unlikely, and if the experiment is fair or unfair. *Remarks/Examples*: The student will represent probabilities as fractions and decimals between 0 and 1 (inclusive), and as percentages between 0% and 100% (inclusive), and verify that the probabilities are reasonable. In 2007 mathematics standards, the concept of probability is introduced for the first time in 7th grade. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.7.P.7.2	Determine, compare, and make predictions based on experimental or theoretical probability of independent or dependent events, *Remarks/Examples*: Experiments could involve or not involve "replacement" of an event. Students must be able to distinguish between independent and dependent events.

	Example: Find the probability of choosing a red marble from a bag of 9 white marbles and 1 red marble, with or without replacement of each drawn marble. Students use manipulatives to obtain experimental results, compare results to mathematical expectations, and discuss the validity of the experiment. *Cognitive Complexity/Depth of Knowledge Rating:* High

GRADE: 8

Big Idea 1: BIG IDEA 1

Analyze and represent linear functions, and solve linear equations and systems of linear equations.

BENCHMARK CODE	BENCHMARK
MA.8.A.1.1	Create and interpret tables, graphs, and models to represent, analyze, and solve problems related to linear equations, including analysis of domain, range, and the difference between discrete and continuous data. *Remarks/Examples*: Example 1: Jan decided to save some money. She already had $25. She received and saved $5 on Friday each week for 8 weeks. Make a table and a graph of the money she would have each week. If she continues with this same savings plan, how much money will she have after 2 years? Is the situation in this problem continuous or discrete? The problem above is technically a discrete problem. A continuous linear function such as y=25+ 5x may be used to fit the data and to solve the problem. If the domain is integers, this is a discrete function. If the domain is all real numbers, this is a continuous function. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.8.A.1.2	Interpret the slope and the x- and y-intercepts when graphing a linear equation for a real-world problem. *Remarks/Examples*: Example: For the example 1in benchmark MA.8.A.1.1, graph the equation y = 5w + 25. Tell why the line "slopes up" by 5 each week.

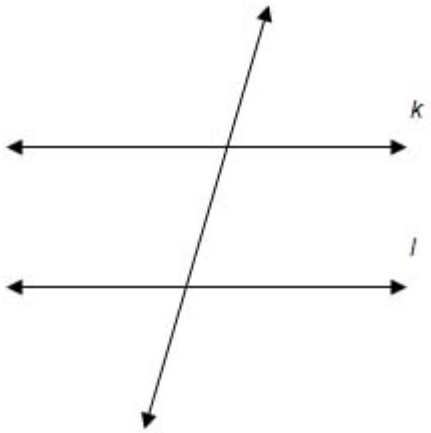
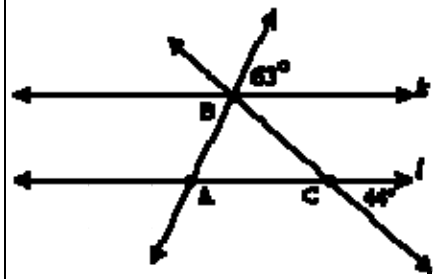
	Also tell why the line crosses the y-axis at 25. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.8.A.1.3	Use tables, graphs, and models to represent, analyze, and solve real-world problems related to systems of linear equations. *Remarks/Examples*: Example 1: A zoo has turtles (each with four legs) and pelicans (each with two legs). There were 29 animals and 78 legs. How many of each type of animal were there? Your final solution should involve principles of equality. Example 2: The students in Mr. Kemp's class ordered T-shirts for the class. They found two different quotes for the cost of the shirts. Company A charges $4 per shirt. Company B charges $75 plus $3 per shirt. 1. The class plans to order 30 shirts. Which company will be a better deal? 2. For what number of T-shirts is the cost the same for both companies? 3. Does the company you chose for question 1 always offer a better deal? Why or why not? Explain your answers. Students should be encouraged to make tables, graphs, and equations and notice the interconnectedness of these representations. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.8.A.1.4	Identify the solution to a system of linear equations using graphs. *Remarks/Examples*: Remarks: Students should recognize that intersecting lines yield a unique solution; parallel lines yield no solution; and coincidental lines yield an infinite number of solutions. Students may use graphing technology to make observations about the effects of slope on the solution of systems of linear equations. Example: Use a graph of the following functions to determine a solution to the system of equations. $y = 5x + 3$ $y = 3x – 9 + 2x$ Example: Jan started with $25 and saved $5 each week. Bill started at the same time with no money and saved $10 per week. Use a graph to determine if or when Bill and Jan will have the same amount of money. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.8.A.1.5	Translate among verbal, tabular, graphical, and algebraic representations of linear functions. *Remarks/Examples*: Example: Jan started with $25 and saved $5 each week. Bill started at the same time with no money and saved $10 per week. Make a table to display the data, write an equation to show the amount of money each person has each week, and graphically display the

	situation. Explain the relationship between different representations of the same data. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.8.A.1.6	Compare the graphs of linear and non-linear functions for real-world situations. *Remarks/Examples*: Students should understand that some situations can be modeled by a linear function and others cannot. Example: Mark had $100 and added $10 to it each year. Mandy put $100 in the bank, earned 10% interest each year on her total amount of money in the bank, and left the interest in the bank account. Make a table of their money for 5 years. Graph the values. Explain why one function is linear and the other one is not. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Big Idea 2: BIG IDEA 2

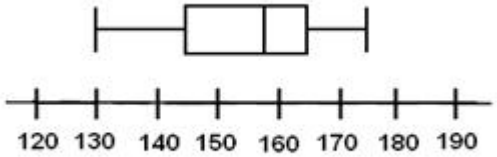
Analyze two- and three-dimensional figures by using distance and angle.

BENCHMARK CODE	BENCHMARK
MA.8.G.2.1	Use similar triangles to solve problems that include height and distances. *Remarks/Examples*: Example 1: At the same time a 10 ft flagpole casts an 8 ft shadow, a nearby tree casts a 40 ft shadow. How tall is the tree? Example 2: A 72-inch tall man casts a shadow that is 96 inches long. At the same time, a nearby crane casts a 52-foot long shadow. How tall is the crane? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.8.G.2.2	Classify and determine the measure of angles, including angles created when parallel lines are cut by transversals. *Remarks/Examples*: Students identify congruent angles, and unique pairings of angles that can be used to determine the measure of missing angles. Example 1: Given that lines *k* and *l* are parallel, determine which angles are vertical, complementary, supplementary, and corresponding.

	 Example 2: Use a map of your town and ask students to identify vertical, complementary, supplementary, and right angles that are formed by the roads. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.8.G.2.3	Demonstrate that the sum of the angles in a triangle is 180-degrees and apply this fact to find unknown measure of angles and the sum of angles in polygons. *Remarks/Examples*: Example 1: "Make a paper triangle and cut off regions around the vertices. Then place the vertices together, meeting at a common point, to see that they form a (approximate) straight angle." Example 2: In the following diagram, line k is parallel to line l. Use properties of angles made when parallel lines are cut by transverse lines to demonstrate that the sum of the three interior angles of a planar triangle is 180 degrees.  Example 3: Determine the sum of the internal angles of a regular hexagon. Investigate whether this sum is the same or different for different hexagons. Explain your findings. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.8.G.2.4	Validate and apply Pythagorean Theorem to find distances in real world situations or between points in the coordinate plane.

	Remarks/Examples: Example 1: You are wrapping a gift for your teacher's birthday. It is a very long and skinny pencil. You want to wrap it in a box so that your teacher can not tell what shape it is. Your friend has a shoe box that measures 10 inches by 7 inches by 5 inches. The pencil is 13 inches long. Will you be able to fit the pencil into the shoe box and close the lid? Justify your answer with mathematics. Example 2: You are sailing your boat to Key West from Pensacola. Key West is 82°W and 25°N, and your boat is 84°W and 29°N. What is the distance from your boat to Key West? Assume 1° change in longitude or latitude is 70 miles.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Big Idea 3: BIG IDEA 3 **Analyze and summarize data sets.**	
BENCHMARK CODE	**BENCHMARK**
MA.8.S.3.1	Select, organize and construct appropriate data displays, including box and whisker plots, scatter plots, and lines of best fit to convey information and make conjectures about possible relationships. *Remarks/Examples*:

	Example: Alfonso's bowling scores are 125, 142, 165, 138, 176, 102, 156, 130, and 142. Make a box-and-whiskers plot of the data. The box and whiskers plot below represents the bowling scores of Anna. Compare the bowling scores of Alfonso and Anna. Who is a better bowler?  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.8.S.3.2	Determine and describe how changes in data values impact measures of central tendency. *Remarks/Examples*: Example: Mrs. Donohue has told her students that she will remove the lowest exam score for each student at the end of the grading period. Sara received grades of 43, 78, 84, 85, 88, 78, and 90 on her exams. What will be the different between the mean, median, and mode of her original grades and the mean, median, and mode of her five grades after Mrs. Donohue removes one grade? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Supporting Idea 4: Algebra

Algebra

BENCHMARK CODE	BENCHMARK
MA.8.A.4.1	Solve literal equations for a specified variable. *Remarks/Examples*: Example 1: Solve the following equation for h: A=bh Example 2: The following equation tells you how much simple interest you will earn if you invest an amount of money (P) at a specified rate (r), for a given amount of time (t): I = Prt. Solve for P. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.8.A.4.2	Solve and graph one- and two-step inequalities in one variable. *Remarks/Examples*: Example: Solve the following inequality for x: 6x-3>10 . Graph the solution set. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Supporting Idea 5: Geometry and Measurement

Geometry and Measurement

BENCHMARK CODE	BENCHMARK
MA.8.G.5.1	Compare, contrast, and convert units of measure between different measurement systems (US customary or metric (SI)) and dimensions including temperature, area, volume, and derived units to solve problems. *Remarks/Examples*: Example 1: Convert 25ºC to degrees Fahrenheit. Example 2: Convert 30 miles per hour to feet per second. $30\frac{miles}{hour} = X\frac{feet}{second}$ Students should not be using only formulas to do this. 1 mile = 5280 feet, and there are 3600 seconds in 1 hour. We may use these equivalencies to substitute feet for miles and seconds for hours. $30\frac{miles}{hour} = 30 \times \frac{5280\,feet}{3600\,seconds} = 44\frac{feet}{seconds}$ Another way to convert units is demonstrated here: $30\frac{miles}{hour}\left(\frac{1hour}{3600\,seconds}\right)\left(\frac{5280\,feet}{1mile}\right) = \frac{30 \times 5280\,feet}{3600\,seconds} = 44\frac{ft}{s}$ *Cognitive Complexity/Depth of Knowledge Rating:* High

Supporting Idea 6: Number and Operations

Number and Operations

BENCHMARK CODE	BENCHMARK
MA.8.A.6.1	Use exponents and scientific notation to write large and small numbers and vice versa and to solve problems. *Remarks/Examples*: Example 1: Write 3,600,000,000 in standard scientific notation. Example 2: Write 0.000 000 000 47 in standard scientific notation.

<table>
<tr><td></td><td>Example 3: Write 6.02×10^{20} without the use of exponents.

<u>Cognitive Complexity/Depth of Knowledge Rating:</u> Low</td></tr>
<tr><td>MA.8.A.6.2</td><td>Make reasonable approximations of square roots and mathematical expressions that include square roots, and use them to estimate solutions to problems and to compare mathematical expressions involving real numbers and radical expressions.

<u>Remarks/Examples</u>:
Example: The formula
$t=\frac{\sqrt{h}}{4}$ represents the time (t) in seconds that it takes an object to fall from a height of h feet. If a ball is dropped from a height of 200 ft, estimate how long it will take to reach the ground.

<u>Cognitive Complexity/Depth of Knowledge Rating:</u> Moderate</td></tr>
<tr><td>MA.8.A.6.3</td><td>Simplify real number expressions using the laws of exponents.

<u>Remarks/Examples</u>:
Example 1: $3^2 \cdot 3^3 = 3\cdot3\cdot3\cdot3\cdot3 = 3^5$
Example 2: Find the value of the expression $4^3 - 3^3$.
Example 3: Simplify the following expression:
$\frac{2^3 3^4 5^4}{4^3 3^3}$

<u>Cognitive Complexity/Depth of Knowledge Rating:</u> Moderate</td></tr>
<tr><td>MA.8.A.6.4</td><td>Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, and irrational numbers) using multi-step and real world problems.

<u>Remarks/Examples</u>:
Example 1: The table shows Mr. Smith's weight during the first 3 months of his diet. If he started his diet at 245 pounds, fill in the following table.

<table>
<tr><td>Month</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Weight</td><td>238</td><td>229</td><td>224</td></tr>
<tr><td>Weight change</td><td></td><td></td><td></td></tr>
</table>

<u>Cognitive Complexity/Depth of Knowledge Rating:</u> High</td></tr>
</table>

GRADE: 912

Body of Knowledge: ALGEBRA

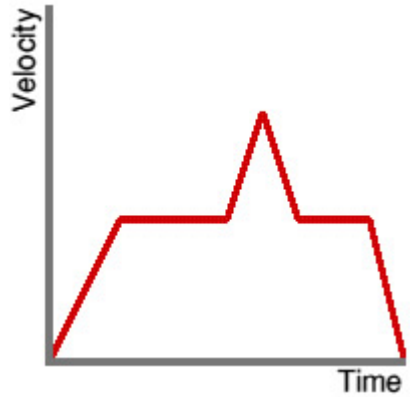
Standard 1: Real and Complex Number Systems

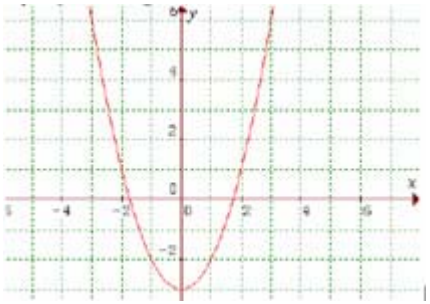
Expand and deepen understanding of real and complex numbers by comparing expressions and performing arithmetic computations, especially those involving square roots and exponents. Use the properties of real numbers to simplify algebraic expressions and equations, and convert between different measurement units using dimensional analysis.

BENCHMARK CODE	BENCHMARK
MA.912.A.1.1	Know equivalent forms of real numbers (including integer exponents and radicals, percents, scientific notation, absolute value, rational numbers, irrational numbers). *Remarks/Examples*: Example: Express 5^{-2} without an exponent. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.1.2	Compare real number expressions. *Remarks/Examples*: Example 1: Which is greater: 2^3 or $\sqrt{49}$? Example 2: Order the following numbers from the smallest to the largest: 3.2, 2.1×10^{-3}, $\sqrt{15}$, -1. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.1.3	Simplify real number expressions using the laws of exponents. *Remarks/Examples*: Example 1: Simplify $5^3 * 5^{11}$. Example 2: Simplify $(5^3)^2$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.1.4	Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, irrational numbers) using multi-step and real-world problems. *Remarks/Examples*: Example 1: If the length of one leg of a right triangle is 6 inches and the length of the hypotenuse is 10 inches, what is the length of the other leg? Example 2: Earth's volume is approximately 1.08×10^{12} km³. Sun's volume is approximately 1.41×10^{18} km³. How many times is the Sun larger than the Earth? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

MA.912.A.1.5	Use dimensional (unit) analysis to perform conversions between units of measure, including rates. *Remarks/Examples*: Example 1: Convert 5 miles per hour to feet per second. Example 2: A sink is leaking 20 milliliters of water per second. How many gallons of water does it leak per day? Example 3: You bought an old car with a 442 cubic inch engine. Your friend has a 7.0 liter engine. Determine which engine is larger by converting 442 cubic inches to liters. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.1.6	Identify the real and imaginary parts of complex numbers and perform basic operations. *Remarks/Examples*: Example: Multiply (7-4i)(10+6i). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.1.7	Represent complex numbers geometrically. *Remarks/Examples*: Example: Plot the point corresponding to 3 - 2i in the complex plane and determine the absolute value of this number. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.1.8	Use the zero product property of real numbers in a variety of contexts to identify solutions to equations. *Remarks/Examples*: Example 1: Solve for x. $(x + 5)(x - 1) = 0$ Example 2: Solve for x. $x^3 - x^2 - 2x + 2 = 0$ Example 3: A ball is kicked and flies through the air according to the following function: h(t)= -16t^2+47t+3, where h is the height of the ball (in feet) and t is the number of seconds after the ball is kicked. At what time, t, does the ball hit the ground after being kicked? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 2: Relations and Functions Draw and interpret graphs of relations. Understand the notation and concept of a function, find domains and ranges, and link equations to functions.	
BENCHMARK CODE	**BENCHMARK**

MA.912.A.2.1	Create a graph to represent a real-world situation. *Remarks/Examples*: Example 1: Conduct an experiment as follows. Take a beverage out of a refrigerator and place it in a warm room. Measure its temperature every two minutes. Plot the temperature of the beverage as a function of time. What does the graph show about the temperature change of this beverage? Example 2: A child walks to school at a steady pace. Plot her distance from home as a function of time. Now plot her distance to the school as a function of time. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.2.2	Interpret a graph representing a real-world situation. *Remarks/Examples*: Example: Jessica is riding a bicycle in a straight line. The graph below shows her speed as it relates to the time she has spent riding. Assign appropriate units to the labels of the axes and insert numbers to the axes. Describe what might have happened to account for this graph.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.2.3	Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions. *Remarks/Examples*: Example 1: Given the relation {(-3, -1), (2, -1), (1, 0), (2, 5)}, determine if the relation can be a function. Example 2: for f(x)=2x+6, find f(3) and find x such that f(x)=10

	Example 3: Given the graph of the relation below, decide if this relation is a function. Explain your reasoning.  <u>*Cognitive Complexity/Depth of Knowledge Rating:*</u> Moderate
MA.912.A.2.4	Determine the domain and range of a relation. <u>*Remarks/Examples*</u>: Example: Determine the domain and range of $f(x) = \sqrt{x}$ so that f(x) is a function. <u>*Cognitive Complexity/Depth of Knowledge Rating:*</u> Moderate
MA.912.A.2.5	Graph absolute value equations and inequalities in two variables. <u>*Remarks/Examples*</u>: Example: Draw the graph of y = |2x - 5| and compare it with the graph of y = 2x - 5. <u>*Cognitive Complexity/Depth of Knowledge Rating:*</u> Moderate
MA.912.A.2.6	Identify and graph common functions (including but not limited to linear, rational, quadratic, cubic, radical, absolute value). <u>*Remarks/Examples*</u>: Example: Graph $f(x) = x^2$, $g(x) = \|x\|, h(x) = \frac{1}{x}$ and $k(x) = \sqrt{x}$ <u>*Cognitive Complexity/Depth of Knowledge Rating:*</u> Moderate
MA.912.A.2.7	Perform operations (addition, subtraction, division, and multiplication) of functions algebraically, numerically, and graphically. <u>*Remarks/Examples*</u>: Example: Let f(x)=7x+2 and g(x)=x². Find f(x)*g(x) <u>*Cognitive Complexity/Depth of Knowledge Rating:*</u> Moderate
MA.912.A.2.8	Determine the composition of functions. <u>*Remarks/Examples*</u>: Example: Let f(x)=x³ and g(x)=x-2. Find f(g(5)) and g(f(x))

	Cognitive Complexity/Depth of Knowledge Rating: Low
MA.912.A.2.9	Recognize, interpret, and graph functions defined piece-wise with and without technology. *Remarks/Examples*: Example: Sketch the graph of $f(x)=\begin{cases} x+2 & x\geq 0 \\ -x^2 & x<0 \end{cases}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.2.10	Describe and graph transformations of functions *Remarks/Examples*: Example: Describe how you would graph $f(x)=-2(x+1)^3-3$ from the graph of $g(x)=x^3$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.2.11	Solve problems involving functions and their inverses. *Remarks/Examples*: Example: Find the inverse of the $f(x)=x^3-1$ function. Sketch the graph of the function and its inverse *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.2.12	Solve problems using direct, inverse, and joint variations. *Remarks/Examples*: Example 1: According to Hooke's Law, the force needed to stretch a spring is directly proportional to the net spring stretch (stretched spring length minus original spring length). If 20 Newtons (N) force results in a net spring stretch of 5 centimeters, what is the net spring stretch achieved when a force of 80N is applied (assuming 80N force does not exceed the spring's stretch limit)? Example 2: On Monday, your drive to work takes 10 minutes and your average speed is 30 mph. On Tuesday, your drive on the same route takes 15 minutes. What is your average speed on Tuesday? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.2.13	Solve real-world problems involving relations and functions. *Remarks/Examples*: Example 1: You and your parents are going to Boston. You will rent a car at Boston's Logan International Airport on a Monday morning and drop the car off in downtown Providence, RI, on the following Wednesday afternoon. Find the rates from two national car companies and plot the costs on a graph. You may choose limited or unlimited mileage plans. Decide which company offers the best

	deal. Explain your answer. Example 2: A cab company charges a fixed flag rate of $20 and $1.40 for every mile covered. Write an expression for the total cab fare as a function of distance driven. Then solve for the total fare after the cab traveled for 36 miles. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 3: Linear Equations and Inequalities Solve linear equations and inequalities.	
BENCHMARK CODE	**BENCHMARK**
MA.912.A.3.1	Solve linear equations in one variable that include simplifying algebraic expressions. *Remarks/Examples*: Example 1: Solve the following equation for x: 3(2x+5) = 10x-3+2x Example 2: Solve the following equation for m: ½m + 2(¾m-1)=¼m+6 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.2	Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality. *Remarks/Examples*: Example 1: Simplify the following expresion and identify the properities used in each step: $(6x^2-5x+1)-2(x^2+3x-4)$ Example 2: Given the following solution identify the properties used to justify each step: 3x+7=2x+1+3x 3x+7=2x + 3x+1 3x+7=5x+1 -2x=-6 x=3 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.3	Solve literal equations for a specified variable. *Remarks/Examples*: Example 1: Solve the following equation for p: q=4p-11. Example 2: Solve the following equation for c: ac=2b + 2c

	Example 3: The area formula for a circle is: $A = \pi r^2$. Solve for *r*.. Solve for . Example 4: The following formula tells you how to convert degrees in Celsius to degrees in Fahrenheit: F=(1.8 x C) +32 Write a formula that will tell how to convert degrees in Fahrenheit to degrees in Celsius. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.4	Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution. *Remarks/Examples*: Example 1: Solve the following inequality for *x* and then graph the solution set on a number line: 7<3x+5<11 Example 2: Solve the following inequality for x in the set {0, 1, 2, 3, 4}: 6x-3>10 Show your work. Example 3: Solve the following inequality for x, explaining each step in your solution: 8x-7≤2x+5 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.5	Symbolically represent and solve multi-step and real-world applications that involve linear equations and inequalities. *Remarks/Examples*: Example 1: You are selling tickets for a play that cost $3 each. You want to sell at least $50 worth. Write and solve an inequality for the minimum number of tickets you must sell. Example 2: An alloy is a metal that contains combinations of different types of metal. A manufacturing company needs to make an alloy that has nickel content between 43% and 47% (based on mass). The company already has an alloy with 50% nickel and another alloy with 40% nickel. They plan to mix them to make the alloy they need. Find the least and greatest mass (in kg) of a 50% nickel alloy that should be mixed with a 40% nickel alloy to end up with 100 kilograms of an alloy containing the required percentage of nickel. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.6	Solve and graph the solutions of absolute value equations and inequalities with one variable. *Remarks/Examples*: Example 1: Given the following equation, solve for x and graph the solution on a number line: |2x=5|=7 Example 2: Given the following inequality, solve for x and graph the solution on a number line: |3x-2|≥5 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.7	Rewrite equations of a line into slope-intercept form and standard form. *Remarks/Examples*: Example 1: Write the following linear equation in standard form 6y = 12 - 5x. Example 2: Write the equation of the line 4x + 3y = 12 in slope-intercept form.

	Cognitive Complexity/Depth of Knowledge Rating: Low
MA.912.A.3.8	Graph a line given any of the following information: a table of values, the x- and y-intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form . *Remarks/Examples*: Example 1: Graph the equation 3x - y = 2. Example 2: Graph the equation y = ½ x + 2 Example 3: Graph the line that contains (3,0) and has a slope of -3/2. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.9	Determine the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line. *Remarks/Examples*: Example: Find the slope and y-intercept of the line described by the equation 4x + 6y = 9. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.10	Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line. *Remarks/Examples*: Example 1: Find an equation of the line through the points (1, 4) and (3, 10). Example 2: Find an equation of the line that goes through the point (5, -2) with a slope of -2 Example 3: Find an equation of the line through the point (1, 4) and perpendicular to y = 3x + 1. Example 4: Find an equation of the line parallel to y = 3x + 2 that passes through the origin. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.11	Write an equation of a line that models a data set, and use the equation or the graph to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change. *Remarks/Examples*: Example 1: As your family is traveling along an interstate, record the odometer reading every 5 minutes. See if a graph of time and distance shows that the relation is approximately linear. If so, write the equation of the line that best fits your data. Predict the time for a journey of 50 miles. What does the slope of the line represent?

	<table><tr><td>Time 0</td><td>Odometer Reading 1 (OR1) ----------</td><td>Initial Odometer Reading (OR0)</td><td>Distance traveled (OR1– OR0) 0</td></tr><tr><td>5 min 10 min</td><td></td><td></td><td></td></tr><tr><td>15 min</td><td></td><td></td><td></td></tr></table> Example 2: You light a candle and record its height in centimeters every minute. The results recorded as (time, height) are (0, 20), (1, 18), (2, 16), (3, 14), (4, 13), (5, 11), (6, 10), (7, 8), (9, 4), and (10, 3). Find the line of best fit to express the candle's height as a function of the time and state the meaning of the slope in terms of the burning candle. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.3.12	Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph. *Remarks/Examples*: Example: On a coordinate plane, graph of the following inequality: $3x+8y\geq24$ Example: Use a spreadsheet to create a line graph of the following function: $y = (3/4)x + 7$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.13	Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology. *Remarks/Examples*: Example 1: Graph $3y - x = 0$ and $2x + 4y = 15$ on the same coordinate system. Determine whether the lines intersect. If so, find the point of intersection. Example 2: Graph the following inequalities and shade the region (if any) on the coordinate plane where both inequalities are true: $y \leq 4$ and $x + y \leq 5$ Example 3: Approximate the solution, if any, for the following system of linear equations:

	$\begin{cases} y = \frac{-1}{4}x + 9 \\ y = 8 \end{cases}$ Example 4: Explain why (4,-3) is a solution to the following system of inequalities: $\begin{cases} y < 3x + 1 \\ x > 2 \end{cases}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.14	Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods. *Remarks/Examples*: Example 1: Solve the following system of equations by substitution: $\begin{cases} y = 2x \\ 2x + 3y = 12 \end{cases}$ Example 2: Graph the solution for the following system of inequalities: $\begin{cases} 3x + 4y < 11 \\ 3x + 2y \geqslant 7 \end{cases}$ Example 3: Solve the following system of equations: $\begin{cases} x - 2y + 3z = 5 \\ x + 3z = 11 \\ 5y - 6z = 9 \end{cases}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.3.15	Solve real-world problems involving systems of linear equations and inequalities in two and three variables. *Remarks/Examples*: Example 1: Each week, you work a total of 20 hours. Some of the 20 hours is spent working at the local bookstore and some spent at the drugstore. You prefer the bookstore and want to work at least 10 more hours at the bookstore relative to the drugstore. Draw a graph to show the possible combinations of hours that you could work.

	Example 2: Let *x* = the amount of liquid (in milliliters) of a product sold by some company. The income (*I*) that the company makes from sales of the liquid can be represented by the equation *I*(x)=10.5x and the expenses (E) for the production of the liquid can be represented by the equation E(x)=5.25x+10,000, where I and E are in dollars. What is the minimum amount of the liquid (in milliliters) that the company must sell to reach the break-even point (the point where income in dollars is equal to expenses in dollars)? Example 3: You need to rent a car to drive from Pensacola to Key West. You will need the car for 7 days. One car rental agency charges $55 per day and $0.06 per mile. Another rental agency charges $65 per day with unlimited mileage. Which rental offer will cost you less? Create a situation where the rental offer in this situation will cost more than the other offer. Explain. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 4: Polynomials

Perform operations on polynomials. Find factors of polynomials, learning special techniques for factoring quadratics. Understand the relationships among the solutions of polynomial equations, the zeros of a polynomial function, the x-intercepts of a graph, and the factors of a polynomial.

BENCHMARK CODE	BENCHMARK
MA.912.A.4.1	Simplify monomials and monomial expressions using the laws of integral exponents. *Remarks/Examples*: Example 1: Simplify $(3a^3)(12a^2)$ Example 2: Simplify: $\frac{15x^7}{3x^5} \quad x \neq 0$ Example 3: Simplify: $(3z^4)^3$ Example 4: Simplify: $(a^0) \quad a \neq 0$ Example 5: Simplify: $(3xy)^3$ Example 6: Simplify:

	$\frac{10}{x^{-4}}$ Example 7: Simplify: $(\frac{a^2b^5}{ab^2})\ a \neq 0,\quad b \neq 0$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.4.2	Add, subtract, and multiply polynomials. *Remarks/Examples*: Example 1: $(4x^3 - 7x + 2) - (x^3 + 4x - 5) = ?$ Example 2: (n+2)(4n-5)=? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.4.3	Factor polynomial expressions. *Remarks/Examples*: Example 1: Factor $36xy^3 + 18xy^4 - 12x^3y^4$ Example 2: Factor $2x^3 - 7x + 3$ Example 3: Factor $4x^3 - 25$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.4	Divide polynomials by monomials and polynomials with various techniques, including synthetic division. *Remarks/Examples*: $\frac{4x^3y^3 + 8xy^4 - 6x^3y^3}{2xy^3}$ Example 1: Simplify Example 2:

	$(2x^3-3x^2+x-6)\div(x^2+2)=?$ Example 3: Use synthetic division to divide $x^3-19x-30$ by x+3. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.5	Graph polynomial functions with and without technology and describe end behavior. *Remarks/Examples*: End behavior may be interpreted as behavior of the function for very large positive or negative(absolutely) independent variables. Example 1: Graph the following equation: $y=x^3-3x^2-x+3$ Example 2: Describe the end behavior for the graph of the following equation $y=-13x^4+7x^3-\frac{2}{3}x^2+8x-7$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.6	Use theorems of polynomial behavior (including but not limited to the Fundamental Theorem of Algebra, Remainder Theorem, the Rational Root Theorem, Descartes' Rule of Signs, and the Conjugate Root Theorem) to find the zeros of a polynomial function. *Remarks/Examples*: Example 1: Given that 4 is a zero of the polynomial $x^3-5x^2-10x+56$, use synthetic divison to find the remaining zeros of the polynomial. Example 2: Use the Remainder Theorem to evaluate $f(x)=6x^3-5x^2+4x-17$ at x=3. Explain your solution method. Example 3: Use the Rational Root Theorem to determine the possible rational roots of the equation $5x^4+3x^3-7x^2+3x+2=0$. Example 4: Use Descartes' Rule of Signs to determine the possible number of positive real zeros and negative real zeros of the following polynomial function: $f(x)=5x^3+3x^2+x-3$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.7	Write a polynomial equation for a given set of real and/or complex roots. *Remarks/Examples*: Example: Find a polynomial equation with the lowest degree possible and with real coefficients that involves the following three roots:

	• 2+i • 3 with a multiplicity of 2 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.8	Describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression with and without technology. *Remarks/Examples*: Example: Use technology to find the solutions of the following equation: $x^3 - 3x^2 - 10x + 24 = 0$. Relate your results to the graph of the function $f(x) = x^3 - 3x^2 - 10x + 24$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.9	Use graphing technology to find approximate solutions for polynomial equations. *Remarks/Examples*: Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest thousandth. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.4.10	Use polynomial equations to solve real-world problems. *Remarks/Examples*: Example: You want to make an open-top box with a volume of 500 square inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Find the possible dimensions of the box. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.11	Solve a polynomial inequality by examining the graph with and without the use of technology. *Remarks/Examples*: Example: Find the solution for $x^3 - 3x^2 - x + 3 < 0$ by graphing the function $f(x) = x^3 - 3x^2 - x + 3$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.4.12	Apply the Binomial Theorem. *Remarks/Examples*: Pascal's triangle is a relevant and interesting structure for examining the Binomial Theorem. Students are expected to know how to use Pascal's triangle in expanding binomials raised to positive integer powers. Example: Expand $(x + 2)^4$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 5: Rational Expressions and Equations Simplify rational expressions and solve rational equations using what has been learned about factoring polynomials.	
BENCHMARK CODE	**BENCHMARK**
MA.912.A.5.1	Simplify algebraic ratios. *Remarks/Examples*: Example: Simplify $\frac{x^2-16}{x^2+4x}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.5.2	Add, subtract, multiply, and divide rational expressions. *Remarks/Examples*: Example: Find the sum of $\frac{x^2-4}{x^3}+\frac{x^2-8}{x^9}$, and tell for what value(s) of x the sum is undefined. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.5.3	Simplify complex fractions. *Remarks/Examples*: Example: Simplify $\frac{\left(\frac{5}{x}\right)}{\left(\frac{1}{x+2}\right)}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.5.4	Solve algebraic proportions. *Remarks/Examples*: Example: Create a tutorial to be posted to the school's Web site to explain how to solve an algebraic proportion for beginning Algebra students. Use $\frac{x+3}{4}=\frac{3x+3}{7}$ as an example. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.5.5	Solve rational equations.

	Remarks/Examples: Example: Solve the following rational equation for n: $\frac{10}{n}+\frac{5}{n^2-4}=\frac{7}{n-2}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.5.6	Identify removable and non-removable discontinuities, and vertical, horizontal, and oblique asymptotes of a graph of a rational function, find the zeros, and graph the function. *Remarks/Examples*: Example: Identify vertical, horizontal, and oblique asymptotes, find the zeros, and graph the following rational functions: $f(x)=\frac{2x+3}{5x-1}$ $g(x)=\frac{x^2-1}{x+1}$ $h(x)=\frac{x}{x^2-4}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.5.7	Solve real-world problems involving rational equations (mixture, distance, work, interest, and ratio). *Remarks/Examples*: Example: It takes Bob 3 hours to paint one side of a house. It takes Joe 2 hours to paint the same side of the house. How long will it take them if they work together? *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 6: Radical Expressions and Equations Simplify and perform operations on radical expressions and equations. Rationalize square root expressions and understand and use the concepts of negative and rational exponents. Add, subtract, multiply, divide, and simplify radical expressions and expressions with rational exponents. Solve radical equations and equations with terms that have rational exponents.	
BENCHMARK CODE	**BENCHMARK**
MA.912.A.6.1	Simplify radical expressions *Remarks/Examples*: Example 1: Simplify $\sqrt{48x^3}$

	Example 2: Simplify $\frac{8}{\sqrt{24}}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.6.2	Add, subtract, multiply, and divide radical expressions (square roots and higher). *Remarks/Examples*: Example 1: Simplify $\sqrt{12}+\sqrt{3x}+7\sqrt{3}$ Example 2: Simplify $\sqrt[4]{3x}-\sqrt[4]{x}-5\sqrt[4]{3x}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.6.3	Simplify expressions using properties of rational exponents. *Remarks/Examples*: Example 1: Simplify $\left(a^{\frac{1}{2}}\right)\left(a^{\frac{1}{3}}\right)$ Example 2: Simplify $\left(\frac{a^{\frac{1}{2}}}{a^{\frac{1}{3}}}\right)$ Example 3: Simplify $\left(a^{\frac{1}{2}}\right)^{\frac{1}{4}}$ Example 4: Simplify $(81)^{-(1/2)}$ Example 5: Simplify $\frac{1}{27^{-(1/3)}}$ *Cognitive Complexity/Depth of Knowledge Rating:* Low

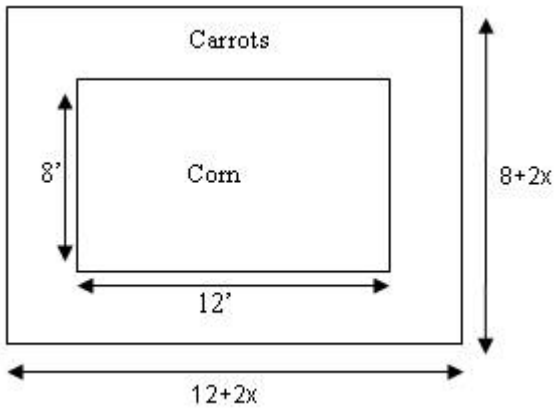
MA.912.A.6.4	Convert between rational exponent and radical forms of expressions. *Remarks/Examples*: Example 1: Rewrite $\sqrt[4]{5^6}$ as 5 to a rational power. Example 2: Rewrite $\sqrt[4]{x^3}$ as x to a rational power. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.6.5	Solve equations that contain radical expressions. *Remarks/Examples*: Example 1: Solve the following equation for x: $\sqrt{x+9} = 9$ Example 2: Solve the following equation for y: $\sqrt{y+9} = 9 - \sqrt{y}$ Example 3: Solve the following equation for z: $z^{\frac{5}{2}} = 32$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 7: Quadratic Equations

Draw graphs of quadratic functions. Solve quadratic equations and solve these equations by factoring, completing the square, and by using the quadratic formula. Use graphing calculators to find approximate solutions of quadratic equations.

BENCHMARK CODE	BENCHMARK
MA.912.A.7.1	Graph quadratic equations with and without graphing technology. *Remarks/Examples*: Example 1: Draw the graph of $y = x^2 - 3x + 2$ Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.7.2	Solve quadratic equations over the real numbers by factoring and by using the quadratic formula. *Remarks/Examples*: Example 1: Solve the following equation for *x*: $x^2 - 3x + 2 = 0$ Example 2: Solve the following equation for *x*: $x^2 - 7x + 9 = 0$

	Cognitive Complexity/Depth of Knowledge Rating: Moderate
MA.912.A.7.3	Solve quadratic equations over the real numbers by completing the square. *Remarks/Examples*: Example 1: Solve the following equation for x: $(x-7)^2 = 64$ Example 2: Solve the following equation for x by completing the square: $x^2 + 6x - 8 = 0$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.7.4	Use the discriminant to determine the nature of the roots of a quadratic equation. *Remarks/Examples*: Example: Use the discriminant to determine whether $x^2 + 6x - 8 = 0$ has distinct real roots. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.7.5	Solve quadratic equations over the complex number system. *Remarks/Examples*: Example: Solve the following equation for x over the set of complex numbers: $x^2 - 2x + 5 = 0$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.7.6	Identify the axis of symmetry, vertex, domain, range and intercept(s) for a given parabola. *Remarks/Examples*: Example: Identify the axis of symmetry, vertex, domain, range, and intercepts for the graph of $y = x^2 + 2x - 3$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.7.7	Solve non-linear systems of equations with and without using technology. *Remarks/Examples*: Example: Find the solution for the following system of equations: $\begin{cases} y = x^2 - 5x + 1 \\ x + y + 2 = 0 \end{cases}$ *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.7.8	Use quadratic equations to solve real-world problems. *Remarks/Examples*:

	Example: You have just planted a rectangular garden of corn in a plot near your home. You want to plant a uniform border of carrots around the rows of corn as shown in the figure below. According to the amount of seeds you have, you need an equal amount of area for corn and carrots. What should the width, x, in feet, of the border be?  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.7.9	Solve optimization problems. *Remarks/Examples*: Example: You have 100 feet of fencing to make three sides of a rectangular area using an existing straight fence as the fourth side. Construct a formula in a spreadsheet to determine the area you can enclose. Use the spreadsheet to make a conjecture about the maximum area possible. Prove (or disprove) your conjecture by solving an appropriate quadratic equation. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.7.10	Use graphing technology to find approximate solutions of quadratic equations. *Remarks/Examples*: Example: Use a graphing calculator to solve the following equation for x to the nearest tenth: $3x^2 - 5x - 1 = 0$ *Cognitive Complexity/Depth of Knowledge Rating:* Low

Standard 8: Logarithmic and Exponential Functions

Understand the concepts of logarithmic and exponential functions. Graph exponential functions, and solve problems of growth and decay. Understand the inverse relationship between exponents and logarithms, and use it to prove laws of logarithms and to solve equations. Convert logarithms between bases, and simplify logarithmic expressions.

BENCHMARK CODE	BENCHMARK
MA.912.A.8.1	Define exponential and logarithmic functions and determine their relationship *Remarks/Examples*: Example: Find the inverse of $f(x) = 2^x$. Identify the domain and range of $f(x)$ and $f^{-1}(x)$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.8.2	Define and use the properties of logarithms to simplify logarithmic expressions and to find their approximate values. *Remarks/Examples*: Example 1: Evaluate the following expression: $\log_3 81$ Example 2: Simplify $2\log x + \log \sqrt{x} + \log y$. Example 3: Find the value of $\log_{10}(10^7)$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.8.3	Graph exponential and logarithmic functions. *Remarks/Examples*: Example 1: Draw the graphs of the functions $f(x) = 2^x$ and $g(x) = 2^{-x}$. Explain their differences and similarities. Example 2: Draw the graphs of the functions $f(x) = \log_2 x$ and $g(x) = 2^x$ and describe their relationship. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.8.4	Prove laws of logarithms. *Remarks/Examples*: Example: Use the fact that $(a^x)(a^y) = a^{(x+y)}$ to show that $\log_a(pq) = \log_a p + \log_a q$ *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.8.5	Solve logarithmic and exponential equations. *Remarks/Examples*: Example 1: Solve the following equation for *x*: $\log_2 x = 5$ Example 2: Solve the following equation for *t*:

	$e^t = 2$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.8.6	Use the change of base formula. *Remarks/Examples*: Example: Write $\log_{10} 75$ as a logarithm of base 2. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.A.8.7	Solve applications of exponential growth and decay. *Remarks/Examples*: Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P(t) is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods? *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 9: Conic Sections

Write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one.

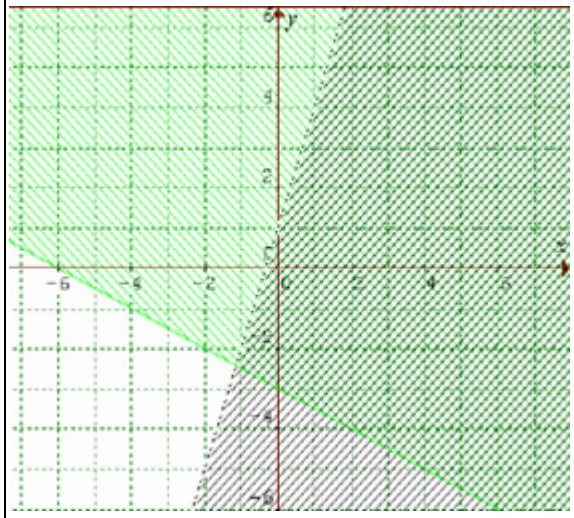
BENCHMARK CODE	BENCHMARK
MA.912.A.9.1	Write the equations of conic sections in standard form and general form, in order to identify the conic section and to find its geometric properties (foci, asymptotes, eccentricity, etc.). *Remarks/Examples*: Example 1: Write the following equation in standard form: $x^2 + y^2 - 10x - 6y - 25 = 0$ Identify the conic and find its foci, asymptotes, and eccentricity as appropriate. Example 2: Write the following equation in standard form: $6x^2 - 36x + 4y^2 - 16y + 70 = 0$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.9.2	Graph conic sections with and without using graphing technology. *Remarks/Examples*: Example: Graph the following conic sections:

	$x^2+y^2+6x-4y-12=0$ $x^2+4y^2=16$ $x=-2y^2+12y-10$ $-\left(\frac{x}{4}\right)^2+\left(\frac{y}{3}\right)^2=1$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.9.3	Solve real-world problems involving conic sections *Remarks/Examples*: Example: The planet Earth orbits the Sun elliptically, with the sun as one of the foci. Given that the length of the major axis of this ellipse is approximately 1.86×10^{8} miles and the eccentricity of the ellipse is about 0.0167, find the smallest distance and the largest distance of Earth from the Sun. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 10: Mathematical Reasoning and Problem Solving

In a general sense, all of mathematics is problem solving. In all of mathematics, use problem-solving skills, choose how to approach a problem, explain the reasoning, and check the results.

BENCHMARK CODE	BENCHMARK
MA.912.A.10.1	Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table. *Remarks/Examples*: Students should work problems where they are required to distinguish relevant from irrelevant information, identify missing information, and either find missing data or make appropriate estimates. Example 1: Fran has scored 16, 23, and 30 points in her last three games. At least how many points must she score in the next game so that her four-game average does not fall below 20 points? Example 2: The swimming pool at Roanoke Park is 24 feet long and 18 feet wide. The park district has determined that they have enough money to put a walkway of uniform width, with a maximum area of 288 square feet, around the pool. How could you find the maximum width of a new walkway? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.A.10.2	Decide whether a solution is reasonable in the context of the original situation.

	Remarks/Examples: Example 1: A student solving the equation $x = \sqrt{x+6}$ comes up with the solution set $\{x \mid x = -2,3\}$. Explain why $\{x \mid x = -2,3\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation. Example 2: A ball is kicked and flies through the air according to the following function: h(t)= -16t^2+47t+3, where h is the height of the ball (in feet) and t is the number of seconds after the ball is kicked. At what time, t, does the ball hit the ground after being kicked? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.A.10.3	Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions). *Remarks/Examples*: Example 1: Alex says x= -1 is the solution to the following system of inequalities. Explain to Alex when x=-1 is a solution and when it is not a solution. y≥-1/2x-3 y<3x+1  Example 2: Is the statement $(a^x)^y = a^{xy}$ true for all x, for some x, or for no x? Example 3: Let c be any constant number different than 5. Which of the following lines will always be parallel to y=2x+5? Explain your answer. a. y= -2x + c b. y= ½ x + c c. y= 2x + c d. y= -½ x + c

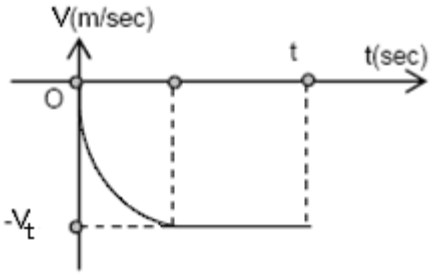
	Cognitive Complexity/Depth of Knowledge Rating: High
MA.912.A.10.4	Use counterexamples to show that statements are false. *Remarks/Examples*: Example 1: Show by an example that the following statement is false: "The product of two complex numbers is never a real number." Example 2: "All quadratic equations have exactly two distinct real roots." Provide a counter example to show that the statement in quotation marks is false. *Cognitive Complexity/Depth of Knowledge Rating:* High

Body of Knowledge: CALCULUS

Standard 1: Limits and Continuity

Develop an understanding of the concept of limit by estimating limits graphically and numerically and evaluating limits analytically. Extend the idea of a limit to one-sided limits and limits at infinity. Use limits to define and understand the concept of continuity, decide whether a function is continuous at a point, and find types of discontinuities. Understand and apply continuity theorems.

BENCHMARK CODE	BENCHMARK
MA.912.C.1.1	Understand the concept of limit and estimate limits from graphs and tables of values. *Remarks/Examples*: Example 1: For $f(x) = \frac{x^3+2x-8}{x-2}$, estimate $\lim_{x \to 1} \frac{x^3+2x-8}{x-2}$ by calculating the function's values for x = 2.1, 2.01, 2.001 and for x = 1.9, 1.99, 1.999. Explain your answer. Example 2: A dog started to chase Kathy from 100 meters away. The dog runs fast so that every minute, the distance between Kathy and the dog is halved. Make a graph that shows the distance between Kathy and the dog in meters versus the time in minutes. Write a function to determine the distance between Kathy and the dog at any given time. Will the dog ever catch Kathy? Write a statement about the distance between Kathy and the dog as the time increases. Example 3: A skydiver free falls from an airplane. The following graph shows the velocity of the skydiver. The air resistance and the gravity are the two forces that affect the velocity of a falling object. Write a paragraph that explains the graph, including but not limited to how the velocity of the skydiver changes as the time increases. You might read about the concept of terminal velocity to make an accurate explanation of the graph.

	 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.1.2	Find limits by substitution. *Remarks/Examples*: Example 1: Find $\lim_{x\to 5}(2x+1)$ Example 2: Find $\lim_{x\to 7}(-3x^5)$ Example 3: Find $\lim_{x\to 0}\frac{e^{x^2}}{3x-4}$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.1.3	Find limits of sums, differences, products, and quotients. *Remarks/Examples*: Example: Find $\lim_{x\to \pi}(sinxcosx+tanx)$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.1.4	Find limits of rational functions that are undefined at a point. *Remarks/Examples*: Example 1: Find $\lim_{x\to 2}\frac{x^2+2x-8}{x-2}$ Example 2: The magnitude of the force between two positive charges, q1 and q2 can be described by the following function: $F(r)=k\frac{q_1q_2}{r^2}$, where *k* is a constant, called Coulomb's constant, and *r* is the distance between the two charges. Find $\lim_{r\to 0}F(r)$.

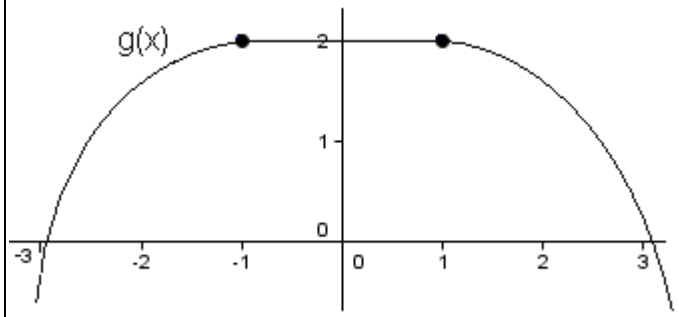
	Interpret the answer in the context of the force between the two charges. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.1.5	Find one-sided limits. *Remarks/Examples*: Example 1: Find $\lim_{x \to 4^-} -\sqrt{4-x}$ Example 2: Find $\lim_{x \to 1^-} \frac{x^2-3x+2}{\lvert x-1 \rvert}$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.1.6	Find limits at infinity. *Remarks/Examples*: Example 1: Find $\lim_{x \to \infty} \frac{x}{x-1}$ Example 2: Find $\lim_{x \to \infty} (2x^3 - 500x^2)$ Example 3: Find $\lim_{x \to -\infty} \frac{x^3 - x + 10}{x^4 - 8}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.1.7	Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. *Remarks/Examples*: Example 1: Find $\lim_{x \to 0} \frac{x^2-3x}{x^2}$ Example 2: Where does the following function have asymptote(s)? Explain your answer. $f(x) = \frac{1}{x^2 - 7x + 10}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

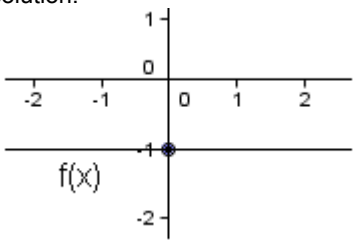
MA.912.C.1.8	Find special limits such as $\lim_{x\to 0}\frac{sinx}{x}$ *Remarks/Examples*: Example: Use a diagram to show that $\lim_{x\to 0}\frac{sinx}{x}$ is equal to 1. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.1.9	Understand continuity in terms of limits. *Remarks/Examples*: Example 1: Show that f(x)=3x + 1 is continuous at x = 2 by finding $\lim_{x\to 2}(3x+1)$ and comparing it with f(2). Example 2: Given that the limg(x) as x approaches to 5 exists, is the statement "g(x) is continuous at x=5" necessarily true? Provide example functions to support your conclusion. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.1.10	Decide if a function is continuous at a point. *Remarks/Examples*: Example: Determine if the function $f(x)=\frac{x^2+2x-8}{x-2}$ can be made continuous by defining the function with a specific value at x=2. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.1.11	Find the types of discontinuities of a function. *Remarks/Examples*: Example: Suppose h(x) = $f(x)=\frac{x^2-5x+6}{x^2-4}$. Identify and categorize any discontinuities in h(x). Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.1.12	Understand and use the Intermediate Value Theorem on a function over a closed interval. *Remarks/Examples*: Example 1: Use the Intermediate Value Theorem to show that $g(x)=x^3+3x^2-9x-2$ has a zero between x = 0 and x = 3. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.1.13	Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval. *Remarks/Examples*: Example: Use the Extreme Value Theorem to decide whether f(x)=tan(x) has a minimum and maximum on the interval $[\frac{-\pi}{4},\frac{\pi}{4}]$. What about on the interval $[-\pi,\pi]$? Explain your reasoning.

	Cognitive Complexity/Depth of Knowledge Rating: Moderate

Standard 2: Differential Calculus

Develop an understanding of the derivative as an instantaneous rate of change, using geometrical, numerical, and analytical methods. Use this definition to find derivatives of algebraic and transcendental functions and combinations of these functions (using, for example, sums, composites, and inverses). Find second and higher order derivatives. Understand and use the relationship between differentiability and continuity. Understand and apply the Mean Value Theorem. Find derivatives of algebraic, trigonometric, logarithmic, and exponential functions. Find derivatives of sums, products, and quotients, and composite and inverse functions. Find derivatives of higher order, and use logarithmic differentiation and the Mean Value Theorem.

BENCHMARK CODE	BENCHMARK
MA.912.C.2.1	Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as an instantaneous rate of change or as the slope of the tangent line. *Remarks/Examples*: Example: Approximate the derivative of $f(x) = x^2$ at x=5 by calculating values of $\frac{f(x+h) - f(x)}{h}$ for values of h that are very close to zero. Use a diagram to explain what you are doing and what the result means. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.2.2	State, understand, and apply the definition of derivative. *Remarks/Examples*: Example 1 (related to the example given in C.2.1):Find $\lim_{h \to 0} \frac{(5+h)^2 - 5^2}{h}$. What does the result tell you? Use the limit given above to determine the derivative function for f(x). In other words calculate f '(x) = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2$. Example 2: For the function g(x), shown on the graph, draw the graph of g'(x) by estimation. Explain how you arrived at your solution.  Example 3: The graph of the function f(x) is given below. Find a function g(x) such that the derivative of g(x) will be f(x). Explain your

	solution.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.2.3	Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions. *Remarks/Examples*: Example 1: Find $\frac{dy}{dx}$ for the function $y = x^2$. Example 2: Find $\frac{dy}{dx}$ for the function y = ln(x). *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.2.4	Find the derivatives of sums, products, and quotients. *Remarks/Examples*: Example 1: Find the derivative of the function f(x)= xcos(x). Example 2: Using the quotient rule for derivatives, show that the derivative of f(x) = tan(x) is f '(x) = $\sec^2(x)$. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.2.5	Find the derivatives of composite functions using the Chain Rule. *Remarks/Examples*: Example 1: Find $f'(x)$ for $f(x) = (x^2 + 2)^2$. Example 2: Find $f'(x)$ for $f(x) = \sin(\frac{1}{x})$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.2.6	Find the derivatives of implicitly-defined functions. *Remarks/Examples*: Example: For the equation $xy - x^2y^2 = 5$, find $\frac{dy}{dx}$ at the point (2, 3).

	Cognitive Complexity/Depth of Knowledge Rating: Moderate
MA.912.C.2.7	Find derivatives of inverse functions. *Remarks/Examples*: Example: Let $f(x) = 2x^3$ and $g(x) = f^{-1}(x)$ find $g'(2)$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.2.8	Find second derivatives and derivatives of higher order. *Remarks/Examples*: Example: Let $f(x) = e^{5x}$. Find $f''(x)$ and $f'''(x)$. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.2.9	Find derivatives using logarithmic differentiation. *Remarks/Examples*: Example 1: Find $\frac{dy}{dx}$ for the following equation: $y = \sqrt{(x+3)^3(x-7)}$. Example 2: Find the derivative of $f(x) = (3x^2+5)^x$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.2.10	Understand and use the relationship between differentiability and continuity. *Remarks/Examples*: Example 1: Let f(x)=1/x. Is f(x) continuous at x = 0? Is f(x) differentiable at x = 0? Explain your answers. Example 2: Is f(x) = IxI continuous at x=0? Is f(x) differentiable at x=0? Explain your answers. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.2.11	Understand and apply the Mean Value Theorem. *Remarks/Examples*: Example 1: Let $f(x) = \sqrt{x}$. On the interval [1, 9], find the value of c such that $\frac{f(9)-f(1)}{9-1} = f'(c)$. Example 2: At a car race, two cars join the race at the same point at the same time. They finish the race in a tie. Prove that some time during the race, the two cars had exactly the same speed. (Hint: Define f(t), g(t), and h(t), where f(t) is the distance that car 1 has traveled at time t, g(t) is the distance that car 2 has travelled at time t, and h(t) = f(t) - g(t).) *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 3: Applications of Derivatives

Apply knowledge about derivatives to find slopes of curves and the related tangent lines. Analyze and graph functions, finding where they are

increasing or decreasing, their maximum and minimum points, their points of inflection, and their concavity. Solve optimization problems, find average and instantaneous rates of change (including velocities and accelerations), and model rates of change. Find slopes and equations of tangent lines, maximum and minimum points, and points of inflection. Solve optimization problems, and find rates of change.

BENCHMARK CODE	BENCHMARK
MA.912.C.3.1	Find the slope of a curve at a point, including points at which there are vertical tangent lines and no tangent lines. *Remarks/Examples*: Example 1: Find the slope of the line tangent to the graph of the equation $y = x^3$ at the point (2, 8). Example 2: Find the slope of the line tangent to the graph of the function $f(x) = \sqrt[3]{(1-x)}$ at x=1. Explain your answer. Example 3: Find the slope of the line tangent to the graph of the function $f(x) = \lvert x^3 - 8 \rvert$ at x=2. Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.2	Find an equation for the tangent line to a curve at a point and a local linear approximation. *Remarks/Examples*: Example 1: Find an equation of the line tangent to the graph of the equation $y = x^3$ at the point (2, 8). Example 2: Use a local linear approximation to estimate the derivative of $f(x) = x^x$ at x=2. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.3	Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of *f* and the sign of *f'*. *Remarks/Examples*: Example 1: For what values of x, is the function $f(x) = \frac{x}{x^2+1}$ decreasing? Example 2: The weight of a new infant baby during the first two months can be modeled by the following function: $w = \frac{1}{4}t^3 + \frac{5}{2}t^2 - \frac{19}{6}t + 8$, w represents weight in pounds, and t represents time in months. When is the infant gaining weight or losing weight during the first two months? Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.4	Find local and absolute maximum and minimum points. *Remarks/Examples*: Example 1: For the graph of the function $f(x) = x^3 - 3x$, find the local maximum and local minimum points of f(x) on [-2, 3].

	Cognitive Complexity/Depth of Knowledge Rating: Moderate
MA.912.C.3.5	Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f". Understand points of inflection as places where concavity changes. *Remarks/Examples*: Example: For the graph of the function $f(x) = x^3 - 3x$, find the points of inflection of f(x) and determine where f(x) is concave upward and concave downward. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.6	Use first and second derivatives to help sketch graphs. Compare the corresponding characteristics of the graphs of f, f', and f". *Remarks/Examples*: Example: Use information from the first and second derivatives to sketch the graph of $f(x) = x^4 + 3x^2 - 2x + 1$. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.3.7	Use implicit differentiation to find the derivative of an inverse function. *Remarks/Examples*: Example: Let $f(x) = 2x^3$ and $g(x) = f^{-1}(x)$ Find g'(x) using implicit differentiation. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.8	Solve optimization problems. *Remarks/Examples*: Example 1: You want to enclose a rectangular field with an area of $5,000m^2$. Find the shortest length of fencing you can use. Example 2: The sum of the perimeters of an equilateral triangle and a square is 20. Find the dimensions of each that will produce the least area. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.9	Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed, and acceleration. *Remarks/Examples*: Example: The vertical distance traveled by an object within the earth's gravitational field (and neglecting air resistance) is given by the equation $x(t) = \frac{1}{2}gt^2 + v_0t + x_0$, where g is the force on the object due to earth's gravity, Vo is the initial velocity, *X*o is the initial height above the ground, t is the time in seconds, and down is the negative vertical direction. Determine the instantaneous speed and the average speed for an object, initially at rest, 3 seconds after it is dropped from a 100m tall cliff. What about 5 seconds after?. Use

	$g = -10\frac{m}{s^2}$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.10	Find the velocity and acceleration of a particle moving in a straight line. *Remarks/Examples*: Example: A bead on a wire moves so that, after t seconds, its distance s cm from the midpoint of the wire is given by $s = 5sin(t - \pi/4)$. Find its maximum velocity and where along the wire this occurs. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.3.11	Model rates of change, including related rates problems. *Remarks/Examples*: Example: One boat is heading due south at 10 mph. Another boat is heading due west at 15 mph. Both boats are heading toward the same point. If the boats maintain their speeds and directions, they will meet in two hours. Find the rate (in miles per hour) that the distance between them is decreasing exactly one hour before they meet. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.3.12	Solve problems using the Newton-Raphson method. *Remarks/Examples*: Example 1: Use three iterations of Newton's method to approximate the zero(s) of $f(x) = x - \cos x$. Example 2: Approximate the zero(s) of the function $f(x) = 1 - 2x^3$ using Newton's Method. Continue until two successive approximations differ by less than 0.001. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 4: Integral Calculus

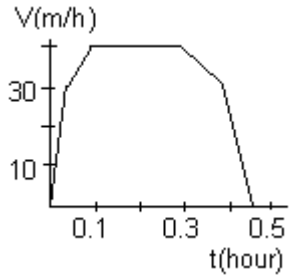
Understand that integration is used to find areas, and evaluate integrals using rectangular approximations. From this, develop the idea that integration is the inverse operation to differentiation — the Fundamental Theorem of Calculus. Use this result to find definite and indefinite integrals, including using the method of integration by substitution. Apply approximate methods, such as the Trapezoidal Rule, to find definite integrals. Define integrals using Riemann sums, use the Fundamental Theorem of Calculus to find integrals using antiderivatives, and use basic properties of integrals. Integrate by substitution, and find approximate integrals.

BENCHMARK CODE	BENCHMARK
MA.912.C.4.1	Use rectangle approximations to find approximate values of integrals. *Remarks/Examples*: Example: Find an approximate value for $\int_0^3 x^2 dx$ using 6 rectangles of equal width under the graph of $f(x) = x^2$ between x=0 and x=3. How did you form your rectangles? Compute this approximation three times using at least three different methods to form the rectangles. *Cognitive Complexity/Depth of Knowledge Rating:* Low

MA.912.C.4.2	Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points. *Remarks/Examples*: Example 1: Find the value of the Riemann Sum over the interval [0, 1] using 6 subintervals of equal width for $f(x) = e^x$ evaluated at the midpoint of each subinterval. Example 2: Estimate $\int_0^{\pi} \sin x \, dx$ using a Riemann midpoint sum with 4 subintervals. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.4.3	Interpret a definite integral as a limit of Riemann sums. *Remarks/Examples*: Example: Find the values of the Riemann sums over the interval [0, 1] using 12 and 24 subintervals of equal width for $f(x) = e^x$ evaluated at the midpoint of each subinterval. Write an expression for the Riemann sums using n intervals of equal width. Find the limit of this Riemann Sums as n goes to infinity. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.4.4	Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval. That is, $\int_a^b f'(x)dx = f(b) - f(a)$ (Fundamental Theorem of Calculus). *Remarks/Examples*: Example: Explain why $\int_4^5 2x dx = 5^2 - 4^2$. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.4.5	Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined. *Remarks/Examples*: Example 1: Using antiderivatives, find $\int_0^3 x^2 dx$ Example 2: Evaluate $\int_1^5 e^x dx$ Example 3: Find

	$\int \sqrt{x}dx$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.4.6	Use these properties of definite integrals: • $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ • $\int_a^b k \cdot f(x)dx = k \int_a^b f(x)dx$ • $\int_a^a f(x)dx = 0$ • $\int_a^b f(x)dx = -\int_b^a f(x)dx$ • $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ • If $f(x) \le g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \le \int_a^b g(x)dx$ *Remarks/Examples*: Example 1: Given $\int_0^3 f(x)dx = 9$ and $\int_{-3}^0 f(x)dx = -9$, find $\int_0^3 5f(x)dx$, $\int_{-3}^3 f(x)dx$, and $\int_0^3 [f'(x) + 2]dx$. Example 2: Evaluate $\int_0^{2\pi} (\sin x + \cos x)dx$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.4.7	Use integration by substitution (or change of variable) to find values of integrals. *Remarks/Examples*: Example: Find $\int x^2(x^3 + 1)^4 dx$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.4.8	Use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values. *Remarks/Examples*:

	Example 1: Use the Trapezoidal Rule with 6 subintervals over [0, 3] for $f(x) = x^2$ to approximate the value of $\int_0^3 x^2 dx$. Example 2: Find an approximation to $\int_{-3}^{0} \sqrt{9 - x^2}\, dx$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 5: Applications of Integration Apply knowledge about integrals to finding velocities from accelerations, solving separable differential equations, and finding areas and volumes. Apply integration to model, and solve problems in physics, biology, economics, etc. Find velocity functions and position functions from their derivatives, solve separable differential equations, and use definite integrals to find areas and volumes.	
BENCHMARK CODE	**BENCHMARK**
MA.912.C.5.1	Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and solving applications related to motion along a line. *Remarks/Examples*: Example 1: A bead on a wire moves so that its velocity (in cm/s), after t seconds, is given by v(t) = 3 cos 3t. Given that it starts 2 cm to the left of the midpoint of the wire, find its position after 5 seconds. Example 2: Carla recorded their car's speed during their trip from school to home. She plotted the data and obtained the following graph. What might the graph for distance versus time look like for their trip to home? Label the axes of your graph and explain why you think it might be a correct representation of the distance versus time for their trip.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.5.2	Solve separable differential equations, and use them in modeling. *Remarks/Examples*: Example 1: Solve and find a general solution to the following differential equation: $\frac{dx}{dy} = 3e^y x^2$. Example 2: A certain amount of money, P, is earning interest continually at a rate of r. Write a separable differential equation to model the rate of change of the amount of money with respect to time.

	Cognitive Complexity/Depth of Knowledge Rating: Moderate
MA.912.C.5.3	Solve differential equations of the form $\frac{dy}{dt} = ky$ as applied to growth and decay problems. *Remarks/Examples*: Example: The amount of a certain radioactive material was 10 kg a year ago. The amount is now 9 kg. When will it be reduced to 1 kg? Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.5.4	Use slope fields to display a graphic representation of the solution to a differential equation, and locate particular solutions to the equation. *Remarks/Examples*: Example: Draw a slope field for $\frac{dy}{dx} = x^2$ and graph the particular solution that passes through the point (2,4). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.5.5	Use definite integrals to find the area between a curve and the x-axis or between two curves. *Remarks/Examples*: Example: Find the area bounded by $y = \sqrt{x}$, y= 0, and x = 2. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.C.5.6	Use definite integrals to find the average value of a function over a closed interval. *Remarks/Examples*: Example 1: Find the average value of y = $y = \sqrt{x}$ over [0, 2]. Example 2: The daytime temperature (in Fahrenheit) in a certain city t hours after 8am can be modeled by the function T= 54 + 15sin(pi*t/12) . What is the average temperature in this city during the time period from 8am to 8pm? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.C.5.7	Use definite integrals to find the volume of a solid with known cross-sectional area, including solids of revolution. *Remarks/Examples*: Example 1: A cone with its vertex at the origin lies symmetrically along the x-axis. The base of the cone is at x = 5 and the base radius is 7. Use integration to find the volume of the cone. Example 2: What is the volume of the solid created when the area between the curves $f(x) = x$ and $g(x) = x^2$ for $0 \le x \le 1$ is revolved around the y-axis? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.C.5.8	Apply integration to model, and solve problems in physical, biological, and social sciences. *Remarks/Examples*: Example: During an acceleration trial, a test vehicle traveling in a straight line has a velocity given by the equation v(t)=sin t , where t is in seconds and velocity is in feet per second. Find the total distance traveled by the test car during the time interval from 0 seconds to 1.5 seconds.

	Cognitive Complexity/Depth of Knowledge Rating: Moderate

Body of Knowledge: DISCRETE MATHEMATICS

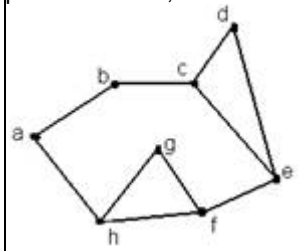
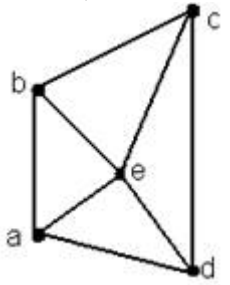
Standard 1: Recursion

Understand and apply recursive methods to solve problems, including the use of finite differences.

BENCHMARK CODE	BENCHMARK
MA.912.D.1.1	Use recursive and iterative thinking to solve problems, including identification of patterns, population growth and decline, and compound interest. *Remarks/Examples*: Example 1: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 15. Example 2: Mary has $1000 at the beginning of 2008. She is going to invest all that money plus $600 every year from now in a certain account that brings in an annual yield of 6.8%. Assume that the interest rate is stable. Let B_0 represents the initial money, B_1 represents the amount of money at the end of 2008 (year1), B2 represents the amount of money at the end of 2009 (year2), and so on. Write a recursive function to find out Mary's money at the end of any given year (year n). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.1.2	Use finite differences to solve problems and to find explicit formulas for recurrence relations. *Remarks/Examples*: Example: Given the set of points {(1,-3), (2.2), (3,13), (4,30), (5,53) }use the method of finite differences to find a polynomial expression that generates these points. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.D.1.3	Use mathematical induction to prove various concepts in number theory (such as sums of infinite integer series, divisibility statements, and parity statements), recurrence relations, and other applications. *Remarks/Examples*: Example 1: Prove that the sum of the first n odd positive integers is n^2. Example 2: Prove that $5^n - 1$ is divisible by 4 for n≥1 and n is integer. Example 3: Prove that for every integer n≥1, $133 \mid (11^{n+1} + 12^{2n-1})$. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 2: Graph Theory

Understand how graphs of vertices joined by edges can model relationships and can be used to solve various problems with relation to directed graphs, weighted graphs, networks, tournaments, transportation flows, matching, and coverage.

BENCHMARK CODE	BENCHMARK
MA.912.D.2.1	Use Euler and Hamilton cycles and paths in graphs to solve routing problems. *Remarks/Examples*: Example 1: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem. Explain your answer. Example 2: A city planner is planning a bus route. She drew the following route, where each vertex represents a bus stop. She wants to make sure that the bus starts from the terminal, vertex a, travels all the roads exactly once and returns back to the terminal. Is this possible? If not, add additional bus stops (vertices) or roads (edges) to make it possible. What is your strategy?  Example 3: A sales person needs to travel to each city shown on the following graph. He wants to start at city a, visit each city exactly one time, and then return to the initial city (city a). Is this possible? If yes, find such a cycle for him.  *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.D.2.2	Use critical path analysis to solve scheduling problems. *Remarks/Examples*: Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.D.2.3	Use graph coloring techniques to solve problems. *Remarks/Examples*: Example: Color a map of the counties of the State of Florida so that no adjacent counties are the same color. What is the minimum

	number of colors needed? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.2.4	Use spanning trees, rooted trees, binary trees, and decision trees to solve problems. *Remarks/Examples*: Example 1: Suppose that you need to identify a fake coin among 8 coins by using a pan balance. The fake coin is lighter than the other seven coins that all weigh the same. What is the minimum number of weighing needed to guarantee that the fake coin is found? Make a decision tree to represent your solution. Solve the same problem by assuming that the fake coin is either lighter or heavier than the other seven coins. Example 2: Suppose that you will have a single elimination chess tournament in your classroom. Draw the graph of this tournament until you have a single winner. What type of a tree is this? If there are n contestants in a single elimination tournament, how many matches will be played? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.2.5	Use bin-packing techniques to solve problems concerning optimizing resource usage. *Remarks/Examples*: Example: Six large crates of electronic equipment are to be shipped to a warehouse. The crates weigh 2,800, 6,000, 5,400, 1,600, 6,800, and 5,000 pounds. Each delivery truck has a capacity of 10,000 pounds. What is the minimum number of trucks needed to send all the crates? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 3: Social Choice

Analyze election data to evaluate different election methods, and use weighted voting techniques to decide voting power within a group. Understand and use fair division techniques to solve apportionment problems.

BENCHMARK CODE	BENCHMARK
MA.912.D.3.1	Use election theory techniques to analyze election data. *Remarks/Examples*: Example: Each student in your class ranks four kinds of fruit drinks from the most preferred to least preferred. Discuss the merits of various methods for deciding on the overall ranking by the class. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.3.2	Use weighted voting techniques to decide voting power within a group. *Remarks/Examples*: Example: A company has 3 stockholders who have different numbers of votes according to their holdings as follows: 4, 3, and 2. The quota that is the number of votes needed to pass a motion is 5. Find the power index of each stockholder. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.3.3	Use fair division techniques to divide continuous objects. *Remarks/Examples*: Example: Find a method for dividing a piece of cake among three people so that each person feels they have received a fair share.

	Cognitive Complexity/Depth of Knowledge Rating: Moderate
MA.912.D.3.4	Use fair division techniques to solve apportionment problems. *Remarks/Examples*: Example: Find the enrollment of seniors, juniors, sophomores, and freshmen at your high school. If there are 20 seats on the Student Council, how should the representatives be apportioned so that the voting power of each class is proportional to its size? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 4: Linear Programming

Understand how to use linear programming and coordinate geometry to solve simple linear optimization problems.

BENCHMARK CODE	BENCHMARK
MA.912.D.4.1	Solve maximal profit/minimal cost problems. *Remarks/Examples*: Example 1: A country store sells GORP to hikers. The MountainClimber mix package contains one pound of peanuts mixed with four pounds of raisins and sells for $9.75. The Tenderfoot mix package contains two pounds of peanuts mixed with three pounds of raisins and sells for $9.50. The center has 60 pounds of peanuts and 120 pounds of raisins available. How many packages of each mix should the center sell to maximize its income? Example 2: A company produces two varieties of widgets —benchmark and deluxe. A benchmark widget takes 3 hours to assemble and 6 hours to finish. A deluxe widget takes 5 hours to assemble and 5 hours to finish. The assemblers can work no more than 45 hours per week and the finishers can work no more than 60 hours per week. The profit is $32 on a Benchmark widget and $40 on a deluxe widget. Find how many of each model should be produced each week to maximize profit. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 5: Game Theory

Understand and use game theory methods to solve strictly determined games and non-strictly determined games.

BENCHMARK CODE	BENCHMARK
MA.912.D.5.1	Use game theory to solve strictly determined games. *Remarks/Examples*: Example: Consider a card game where John gets a 4 of Hearts and a 5 of Clubs, and Susan gets a 3 of Clubs and a 6 of Hearts. The players each show one card simultaneously. The player who shows the card of larger value receives the sum of the numbers on the two cards shown. Set up the game matrix and find the optimal strategy and the value of the game. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.D.5.2	Use game theory to solve non-strictly determined games. *Remarks/Examples*: Example: In the game “Two-Finger Morra,” each of two players shows either one or two fingers. If the total number of fingers shown is

	even, Player A collects a dollar for each finger shown from Player B. If the total number of fingers is odd, Player A pays $3 to Player B. Set up the game matrix and find the optimal strategy and the value of the game. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 6: Logic

Develop an understanding of the fundamentals of propositional logic, arguments, and methods of proof.

BENCHMARK CODE	BENCHMARK
MA.912.D.6.1	Use truth tables to determine truth values of propositional statements. *Remarks/Examples*: Students develop truth tables involving conjunctive, disjunctive, negation, conditional, and biconditional statements and use these truth tables to solve problems. Example 1: What is the truth value of the following statement? If the Earth is flat, then arsenic is healthy. Example 2: Show that $(p \rightarrow q) \equiv (\sim p \vee q)$. Example 3: Three students A, B, C owned a pet for some time and then disagreed as to who owns the animal. It is not certain that the responsibility of the ownership is desired. Each student makes one true and one false statement: Student A claims -It is C's pet -I can make no claim to it Student B claims -C has no right to it -It is A's pet Student C claims -It is my pet -Student B's second statement is false. Which student does the pet belong to? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.6.2	Find the converse, inverse, and contrapositive of a statement *Remarks/Examples*: Example: Determine the inverse, converse and contrapositive of the statement, “If it is Thursday, there will be rain.” *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

MA.912.D.6.3	Determine whether two propositions are logically equivalent. *Remarks/Examples*: Example: Determine whether the propositions $\sim(p \vee q)$ and $(\sim p \wedge \sim q)$ are logically equivalent. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.6.4	Use methods of direct and indirect proof and determine whether a short proof is logically valid. *Remarks/Examples*: Example: If somebody argues, “If it’s Thursday, it is raining.” along with “It is raining” implies that "it is Thursday.", is this a valid or invalid argument? Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.6.5	Identify and give examples of : • undefined terms; • axioms; • theorems; • inductive and deductive proofs; and, • inductive and deductive reasoning. *Remarks/Examples*: Example 1: Do you prove axioms from theorems or theorems from axioms? Example 2: What type of reasoning are you using when you look for a pattern? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.6.6	Construct logical arguments using laws of detachment (modus ponens), syllogism, tautology, and contradiction; judge the validity of arguments, and give counterexamples to disprove statements. *Remarks/Examples*: Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.D.6.7	Use applications of the universal and existential quantifiers to propositional statements. *Remarks/Examples*: Example: Use predicate logic formulas to write the following phrases: *Only dogs bark. *Everyone has a father. *If a number is an integer, then it is a rational number. *Cognitive Complexity/Depth of Knowledge Rating:* Low

Standard 7: Set Theory

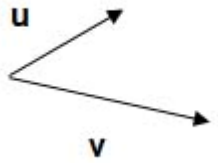
Operate with sets, and use set theory to solve problems.	
BENCHMARK CODE	**BENCHMARK**
MA.912.D.7.1	Perform set operations such as union and intersection, complement, and cross product. *Remarks/Examples*: Example: Let A={1,2,3} and B={2,4,5} be two sets in universe U={1,2,3,4,5,6}. Find the union of A and B and the complement of B. Find AXB. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.7.2	Use Venn diagrams to explore relationships and patterns and to make arguments about relationships between sets. *Remarks/Examples*: Example: Use a Venn diagram to give an argument that the intersection of A and B is a subset of the union of A and B. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 8: Matrices Understand how matrices can be used to store and organize data and to solve systems of equations. Use matrices to solve Markov chain problems that link present events to future events using probabilities.	
BENCHMARK CODE	**BENCHMARK**
MA.912.D.8.1	Use matrices to organize and store data. Perform matrix operations (addition, subtraction, scalar multiplication, multiplication) *Remarks/Examples*: Example 1: Central High School offers three different styles of class rings — benchmark, classic, and deluxe. Each style is available in a girl's ring and a boy's ring. Make up your own data to show how many of each variety was sold and store the data in a matrix with rows and columns labeled. Example 2: For the following matrices perform the indicated operation, if possible: $A=\begin{bmatrix}2 & -1 & 5\\ 3 & 0 & 4\end{bmatrix}$ $B=\begin{bmatrix}-4 & 0 & 0\\ -1 & 1 & 7\end{bmatrix}$ $C=\begin{bmatrix}3 & -1\\ -5 & 8\\ 2 & 0\end{bmatrix}$ B-A , A+C, AC, CA, AB *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.8.2	Use matrix operations to solve problems. *Remarks/Examples*: Example: Suppose the rings in Example 1 for the benchmark MA.921.D.8.1 cost $90, $120, and $135 for the girls' rings and $110, $140, and $165 for the boys' rings. Display this information in a matrix and find the total revenue from the sale of girls' rings and from

	the sale of boys' rings. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.8.3	Use row-reduction techniques to solve problems. *Remarks/Examples*: Example: Solve this system of equations using an augmented matrix and row reduction: x - 2y + 3z = 5 x + 3z = 11 5y - 6z = 9 *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.8.4	Find the inverse of a matrix, and use the inverse to solve problems with and without the use of technology. *Remarks/Examples*: Example: Solve the system of equations in Example 1 for the benchmark MA.912.D.8.3 using an inverse matrix. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.D.8.5	Use determinants of 2 x 2 and 3 x 3 matrices as well as higher order matrices with and without the use of technology. *Remarks/Examples*: Example 1: Explain why a square matrix is invertible if and only if its determinant is non-zero. Example 2: Use Cramer's rule to solve a system of equations such as x - 2y + 3z = 5 x + 3z = 11 5y - 6z = 9 *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.8.6	Use matrices to solve Markov chain problems that link present events to future events using probabilities. *Remarks/Examples*: Example: Ms. Johnson has observed John's grade in her Algebra II class for a long time. It seems that when John gets an A on a quiz, the probability he would get an A, B, C, D, F on the next quiz will be 5/10, 2/10, 2/10, 1/10, 0, respectively. When he gets B, the probability he would get an A, B, C, D, F on the next quiz will be 2/10, 5/10, 2/10, 1/10, 0, respectively. When he gets a C, the probability he would get an A, B, C, D, F on the next quiz will be 1/10, 2/10, 5/10, 1/10, 1/10, respectively. When he gets a D, the probability he would get an A, B, C, D, F on the next quiz will be 1/10, 1/10, 2/10, 5/10, 1/10, respectively. When he gets an F, the probability he would get an A, B, C, D, F on the next quiz will be 1/10, 1/10, 1/10, 2/10, 5/10, respectively. John got a B today on a quiz in Ms. Johnson's class. What is the probability he would get an A after three quizzes? *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 9: Vectors

Recognize vectors in both two- and three-dimensions. Recognize that vectors are represented geometrically and algebraically. Perform basic operations on vectors, including addition, scalar multiplication, dot product, and cross product. Solve problems using vectors.

BENCHMARK CODE	BENCHMARK
MA.912.D.9.1	Demonstrate an understanding of the geometric interpretation of vectors and vector operations including addition, scalar multiplication, dot product, and cross product in the plane and in three-dimensional space. *Remarks/Examples*: Example 1: Vectors u and v are shown below. Use these vectors to sketch 2u + 3v.  Example 2: Find the magnitude and direction of (3i+4j), where i is along x axis and j is along y axis. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.9.2	Demonstrate an understanding of the algebraic interpretation of vectors and vector operations including addition, scalar multiplication, dot product, and cross product in the plane and in three-dimensional space. *Remarks/Examples*: Example: If u = (3, 1) and v = (-3, 2) , find the measure of the angle between vectors u and v. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.9.3	Use vectors to model and solve application problems. *Remarks/Examples*: Example: The vector $\vec{d}$ represents the displacement of a wagon that is pulled with the force $\vec{F}$ .The work done in moving the wagon in the direction of $\vec{d}$ is defined to be the component of $\vec{F}$ in the direction of $\vec{d}$ times the distance the wagon moves. Show that this definition can be written as , $\left\|\vec{W}\right\| = \left\|\vec{F}\right\|\cos(\alpha)\left\|\vec{d}\right\|$ where α= the angle between $\vec{F}$ and $\vec{d}$. And find the work done if $\vec{F}$ = (10,3) and $\vec{d}$ = (25,0). *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 10: Parametric Equations Use parametric equations in two dimensions to model time dependant situations, and convert parametric equations to rectangular coordinates and vice-versa.	
BENCHMARK CODE	**BENCHMARK**
MA.912.D.10.1	Sketch the graph of a curve in the plane represented parametrically, indicating the direction of motion. *Remarks/Examples*: Example: Sketch the graph of the curve with parametric equations x=5-3t, y=-2 + t, and indicate the direction of motion as t increases. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.10.2	Convert from a parametric representation of a plane curve to a rectangular equation and vice-versa. *Remarks/Examples*: Example 1: A curve has parametric representation $x = 2 - 3t, y = 4 + t^2$. Find an equation for the curve in rectangular coordinates. Example 2: Find a parametric representation for the ellipse with the rectangular equation $\frac{x^2}{4} + \frac{y^2}{25} = 1$. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.10.3	Use parametric equations to model applications of motion in the plane. *Remarks/Examples*: Example: Suppose an object moving at constant velocity is at the point A(5,3) when time t=0 seconds, and at point B(-4, 15) when t=3 seconds. Find the velocity and speed of the object, and parametric equations for the motion of the object. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 11: Sequences and Series Define and use arithmetic and geometric sequences and series.	
BENCHMARK CODE	**BENCHMARK**
MA.912.D.11.1	Define arithmetic and geometric sequences and series. *Remarks/Examples*: Example: An investment doubles each decade. If the principal was $1000.00, write the sequence that shows the amount for each of four decades. Is this sequence arithmetic or geometric? Why? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.11.2	Use sigma notation to describe series. *Remarks/Examples*: Example: Using the sigma notation, describe the total area of a set of 12 rectangles. Each of the rectangles has a width of 2 units. The first has a height of 5 units and the height of each successive rectangle is 2 units more than the previous one.

	Cognitive Complexity/Depth of Knowledge Rating: Low
MA.912.D.11.3	Find specified terms of arithmetic and geometric sequences. *Remarks/Examples*: Example: A decorative brick wall is designed with one brick on the top row and each row below the top containing two more bricks than the row above. How many bricks are in the 20th row? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.D.11.4	Find partial sums of arithmetic and geometric series, and find sums of infinite convergent geometric series. Use Sigma notation where applicable. *Remarks/Examples*: Example 1: A decorative brick wall is designed with one brick on the top row and each row below the top containing two more bricks than the row above. How many bricks are needed to make the wall 50 rows high? Example 2: A ball is dropped from a height of 6 feet. It bounces back up to a height of 4 feet, falls back to the ground and continues bouncing. If each bounce is 2/3 the height of the previous bounce, find the total vertical distance traveled by the ball. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.D.11.5	Explore and use other sequences found in nature such as the Fibonacci sequence and the golden ratio. *Remarks/Examples*: Example: Determine the first ten terms of the Fibonacci sequence. Calculate the ratio of the two adjacent terms such as the ratio of the second term to the first term, third term to the second term, fourth term to the third term, and so on. What would happen to the ratio between two adjacent terms as the sequence proceeds? *Cognitive Complexity/Depth of Knowledge Rating:* High

Body of Knowledge: FINANCIAL LITERACY

Standard 1: Simple and Compound Interest

Simple and Compound Interest

BENCHMARK CODE	BENCHMARK
MA.912.F.1.1	Explain the difference between simple and compound interest. *Remarks/Examples*: Example: Compare the similarities and differences for calculating the final amount of money in your savings account based on simple interest or compound interest. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.1.2	Solve problems involving compound interest. *Remarks/Examples*: Example: Find the amount of money on deposit at the end of 5 years if you started with $500 and it was compounded quarterly at 6 %

	interest. Example: Joe won $25,000 in the lottery. How many years will it take at 6% interest compounded yearly for his money to double? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.1.3	Demonstrate the relationship between simple interest and linear growth. *Remarks/Examples*: Example: Find the account balance at the end of each month for a 5 month span for $1500 @ 3 % interest based on simple interest for 1 year. Graph this scenario and explain if this is a linear or exponential problem. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.1.4	Demonstrate the relationship between compound interest and exponential growth. *Remarks/Examples*: Example: Using an exponential function, find the account balance at the end of 4 years if you deposited $1300 in an account paying 3.5% interest compounded annually. Graph the scenario. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 2: Net Present and Net Future Value (NPV and NFV)

Net Present and Net Future Value (NPV and NFV)

BENCHMARK CODE	BENCHMARK
MA.912.F.2.1	Calculate the future value of a given amount of money with and without technology. *Remarks/Examples*: Example: Suppose you have $750 on January 1, 2007. If you deposit this in an account paying 5% interest, compounded quarterly, how much money will be in the account on January 1, 2012? Example: Suppose you deposit $400 into an account at the beginning of each year, starting Jan 1, 2007. If the account pays 6% interest, compounded annually, how much will be in the account at the end of 5 years? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.2.2	Calculate the present value of a certain amount of money for a given length of time in the future with and without technology. *Remarks/Examples*: Example: A five year, zero-coupon bond pays 5% annual interest, and has a face value of $1,000. If the bond matures on Dec 31, 2010, what was the original purchase price of the bond? Example: Find the present value of an annuity paying $500 per year for 10 years at 6% annual interest. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.2.3	Use a consumer price index to express dollars in constant terms with and without technology. *Remarks/Examples*: Example: The U.S. Consumer Price Index for January 2000 was 168.8, and in January 2006 was 198.3. If a worker was making a monthly salary of $2500 in January 2000, how much would (s)he need to earn in January 2006 to keep pace with inflation? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.2.4	Calculate the present value of an income stream with and without technology.

	Cognitive Complexity/Depth of Knowledge Rating: Moderate

Standard 3: Loans and Financing

Become familiar with and describe the advantages and disadvantages of short-term purchases, long-term purchases, and mortgages.

BENCHMARK CODE	BENCHMARK
MA.912.F.3.1	Compare the advantages and disadvantages of using cash versus a credit card. *Remarks/Examples*: Example: Compare paying for a tank of gasoline in cash or paying with a credit card over a period of time. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.3.2	Analyze credit scores and reports. *Remarks/Examples*: Example: Explain how each of the following categories affects a credit score: 1) past payment history, 2) amount of debt, 3) public records information, 4) length of credit history, and 5) the number of recent credit inquiries. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.3	Calculate the finance charges and total amount due on a credit card bill. *Remarks/Examples*: Example: Calculate the finance charge each month and the total amount paid for 5 months if you charged $500 on your credit card but you can only afford to pay $100 each month. Your credit card has a monthly periodic finance rate of .688% and an annual finance rate of 8.9%. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.F.3.4	Compare the advantages and disadvantages of deferred payments. *Remarks/Examples*: Example: Compare paying on a college loan between a Stafford loan or a PLUS loan two years after graduation *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.3.5	Calculate deferred payments. *Remarks/Examples*: Example: You want to buy a sofa that cost $899. Company A will let you pay $100 down and then pay the remaining amount over 3 years at 22% interest. Company B will not make you pay a down payment and they will defer payments for one year. However, you will accrue interest at a rate of 20 % interest during that first year. Starting the second year you will have to pay the new amount for 2 years at a rate of 26 % interest. Which deal is better and why? Calculate the total amount paid for both deals. Example: An electronics company advertises that you don't have to pay anything for 2 years. If you bought a big screen TV for $2999 on January 1st what would your balance be two years later if you haven't made any payments assuming an interest rate of 23.99%? What would your monthly payments be to pay the TV off in 2 years? What did the TV really cost you? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.6	Calculate total cost of purchasing consumer durables over time given different down payments, financing options, and fees. *Remarks/Examples*: Example: Find the actual cost of a car and interest charged with a showroom price of $15,999, down payment of $1,600, rate of interest

	of 12%, and 30 monthly payments. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.7	Calculate the following fees associated with a mortgage: • discount points • origination fee • maximum brokerage fee on a net or gross loan • documentary stamps • prorated expenses (interest, county and/or city property taxes, and mortgage on an assumed mortgage) *Remarks/Examples*: Example: 1) Calculate the total amount of fees on a $230,000 mortgage if the lender: charges 2 points and a 0.5% origination fee. 2) Calculate the maximum brokerage fee on a net loan of $184,000, 3) A seller has agreed to pay the Documentary Stamps on a property worth $150,000 (selling price). The purchaser is responsible for the Documentary Stamps on the $75,000 mortgage being assumed and the new $25,000 second mortgage. Calculate all applicable amounts. 4) A $185,340 loan carries at a 5.625% annual interest rate. Using the 365 day method, how much interest would a buyer owe for the 22 days remaining for a May closing. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.F.3.8	Substitute to solve a variety of mortgage formulas, including but not limited to Front End Ratio, Total Debt-to-Income Ratio, Loan-to-Value Ratio (LTV), Combined Loan-to-Value Ratio (CLTV), and Amount of Interest Paid Over the Life of a Loan. *Remarks/Examples*: Example: Mr. Lindsey purchased a home for $129,000. It was appraised at $95,000. He was assuming a $52,000 first mortgage, and he obtained a 2nd mortgage for the lenders maximum CLTV of 90%. What would be the amount of the down payment? Example: Calculate the interest scheduled to be paid over the life of a $190,000 mortgage loan with a term of 30 years and fixed monthly payment of $1250.50. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.F.3.9	Calculate the total amount to be paid over the life of a fixed rate loan. *Remarks/Examples*: Example: Calculate the total amount to be paid for a $275,000 loan at 5.75% interest over 30 years *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.10	Calculate the effects on the monthly payment in the change of interest rate based on an adjustable rate mortgage. *Remarks/Examples*: Example: You would like to borrow $245,000 using a 30-year, 1-year ARM indexed to the 1-year Treasury security with a 2.75 percent margin and 2/6 caps (2 percent per year and 6 percent lifetime). The initial interest rate on this loan is 2.75 percent. The lender is charging you 1.50 points and $1,200 in miscellaneous fees to close the loan. a) What is the initial payment on this mortgage? b) If the 1- year Treasury security is yielding 2.25 percent at the first adjustment date, what is your payment on this loan during the

	second year? c) Suppose that the 1-year Treasury is yielding 2.75 percent at the second adjustment date. What is the new payment on this loan during the third year? d) Assuming that you pay of the loan at the end of the third year, what yield did the lender earn on this loan? Now resolve all four parts of the last problem assuming that the loan has a 20 percent payment cap instead of 2/6 interest rate caps. a) What is the initial payment on this mortgage? b) If the 1- year Treasury security is yielding 2.25 percent at the first adjustment date, what is your payment on this loan during the second year? c) Suppose that the 1-year Treasury is yielding 2.75 percent at the second adjustment date. What is the new payment on this loan during the third year? d) Assuming that you pay of the loan at the end of the third year, what yield did the lender earn on this loan? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.11	Calculate the final pay out amount for a balloon mortgage. *Remarks/Examples*: Example: If you have a 5-year balloon mortgage with a 15 year amortization schedule, a rate of 6.5%, and a $100,000 loan what would the remaining balance be after the end of the fifth year? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.12	Compare the cost of paying a higher interest rate and lower points versus a lower interest rate and more points. *Remarks/Examples*: Example: Assuming all of the following were originally 15 year mortgages, which fixed rate mortgage cost the mortgagor the least? a) 7.375% interest + 0 points paid off in 10 years b) 7.375% interest + 0 points paid off in 7 years c) 7 % interest + 3 points paid off in 10 years d) 7 % interest + 3 points paid off in 7 years *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.13	Calculate the total amount paid for the life of a loan for a house including the down payment, points, fees, and interest. *Remarks/Examples*: Example: Calculate the total amount paid for a $100,000 house with a 15 year fixed rate loan at 5.65% if the mortgagor pays a $25,000 down payment; 2 points; 1% origination fee; maximum brokerage fee on a net loan; and State Documentary Stamps on the deed at a tax rate of $.70 per $100, the mortgage note at a tax rate of $.35 per $100, a and Intangible Tax at a rate of .002. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.14	Compare the total cost for a set purchase price using a fixed rate, adjustable rate, and a balloon mortgage. *Remarks/Examples*: Example: Find the total cost for a $225,000 mortgage for the following options: a) 30 year fixed rate mortgage with a rate of 6.35 % b) 3/1 ARM with a rate of 6.75% with a maximum adjustment of 2 points per year with a cap of 6 points for 30 years c) 10 year balloon mortgage with a 30 year amortization schedule with a rate of 5.5%

	Next describe the benefits and detriments of each mortgage option. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.15	Interpret the legal description using the metes and bounds; lot and block (plat); government survey; and monument methods. *Remarks/Examples*: Example: Given an unmarked parcel of land and an accurate point of beginning POB) draw in the boundary lines using the given angles and distances. Example: Find a specific lot on a plot. Example: Find a specific range on a government survey. Example: Write a legal description for a specific piece of property using natural topographical features (monument method). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.16	Estimate real property value using the sales comparison approach, cost-depreciation approach, or the income capitalization approach. *Remarks/Examples*: Example: Use the cost-depreciation approach to estimate the real property value of a given home at current builders' market cost per square foot. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.3.17	Compare interest rate calculations and annual percentage rate calculations to distinguish between the two rates. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 4: Individual Financial Planning

Individual Financial and Investment Planning

BENCHMARK CODE	BENCHMARK
MA.912.F.4.1	Develop personal budgets that fit within various income brackets. *Remarks/Examples*: Example: Develop a budget worksheet that includes typical expenses such as housing, transportation, utilities, food, medical expenses, and miscellaneous expenses. Add categories for savings toward your own financial goals, and determine the monthly income needed, before taxes, to meet the requirements of your budget. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.2	Explain cash management strategies including debit accounts, checking accounts, and savings accounts. *Remarks/Examples*: Example: Explain the difference between a checking account and a savings account. Why might you want to have both types of accounts? Why might you want to have only one or the other type? Why is it rare to find someone who has a savings account but no checking account? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.3	Calculate net worth.

	Remarks/Examples: Example: Jose is trying to prepare a balance sheet for the end of the year. His balances and details for the year are given in the table below. Write a balance sheet of Jose's liabilities and assets, and compute his net worth. *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.F.4.4	Establish a plan to pay off debt. *Remarks/Examples*: Example: Suppose you currently have a balance of $4500 on a credit card that charges 18% annual interest. What monthly payment would you have to make in order to pay off the card in 3 years, assuming you do not make any more charges to the card? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.4.5	Develop and apply a variety of strategies to use tax tables, and to determine, calculate, and complete yearly federal income tax. *Remarks/Examples*: Example: Suppose that Joe had income of $40,000 in 2005, and had various deductions totaling $6,240. If Joe filed as a single person, how much income tax did he have to pay that year? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.6	Compare different insurance options and fees. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.4.7	Compare and contrast the role of insurance as a device to mitigate risk and calculate expenses of various options. *Remarks/Examples*: Example: Explain why a person might choose to buy life insurance. Are there any circumstances under which one might not want life insurance? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.8	Collect, organize, and interpret data to determine an effective retirement savings plan to meet personal financial goals. *Remarks/Examples*: Example: Investigate historical rates of return for stocks, bonds, savings accounts, mutual funds, as well as the relative risks for each type of investment. Organize your results in a table showing the relative returns and risks of each type of investment over short and long terms, and use these data to determine a combination of investments suitable for building a retirement account sufficient to meet anticipated financial needs. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.9	Calculate, compare, and contrast different types of retirement plans, including IRAs, ROTH accounts, and annuities. *Remarks/Examples*: Example: Suppose you put $5000 per year into an IRA for 40 years. If the account pays 6% per year interest, how much would you have at the end of the 40 years? If, at that time, you are in the 15% income tax bracket, how much would this be after taxes? Suppose that, instead, you paid the tax each year on the $5000 at your current rate of 28% and put the remaining funds in a ROTH account paying 6% interest. How much would you then have after 40 years? Which appears to be the better option? What are some of the risks of deferring tax payments until retirement? Example: Explain the difference between an Individual Retirement Account (IRA) and a ROTH account.

	Why might somebody choose to put retirement funds in a ROTH account rather than an IRA? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.4.10	Analyze diversification in investments. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.4.11	Purchase stock with a set amount of money, and follow the process through gains, losses, and selling. *Remarks/Examples*: Example: At the beginning of the year, Mary invests $3000, buying $1500 of Stock A at $30 per share, $1000 of Stock B at $40 per share, and putting $500 in a money market account paying 5% interest. At the end of the year, stock A is priced at $34 per share, and stock B is priced at $38 per share. What is the overall rate of return for the year on Mary's investments? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.12	Compare and contrast income from purchase of common stock, preferred stock, and bonds. *Remarks/Examples*: Example: Explain the difference between common and preferred stock. What are some reasons people might choose common stock over preferred stock? Which type of stock is more prevalent in the market today? Example: Compare corporate bonds, government bonds, and common stock as investments with respect to the following attributes: rates of return, price risk, default risk, and taxability of earnings *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.F.4.13	Given current exchange rates be able to convert from one form of currency to another. *Remarks/Examples*: Example: Suppose you are traveling in Europe, and while there you withdraw 150 Euros to pay for expenses. If the exchange rate at the time was $1.27 per Euro, how much money (in dollars) was charged to your bank account? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.F.4.14	Use data to compare historical rates of return on investments with investment claims to make informed decisions and identify potential fraud. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 5: Economic Concepts

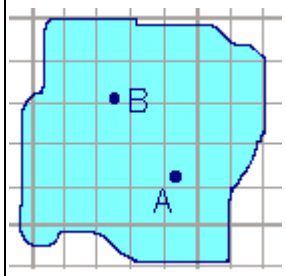
Economic Concepts

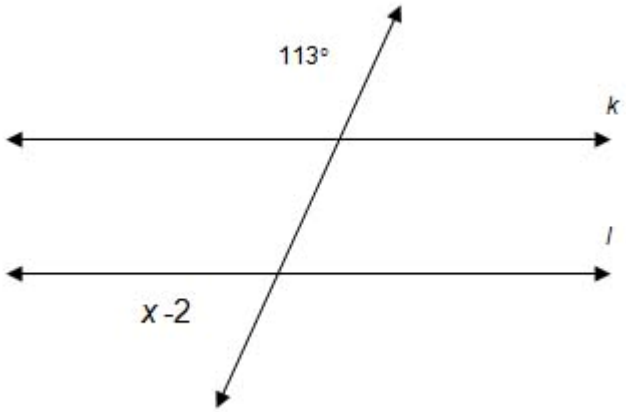
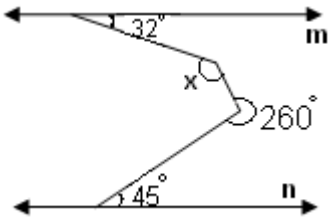
BENCHMARK CODE	BENCHMARK
MA.912.F.5.1	Demonstrate how price and quantity demanded relate, how price and quantity supplied relate, and how price changes or price controls affect distribution and allocation in the economy. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.F.5.2	Use basic terms and indicators associated with levels of economic performance and the state of the economy. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Body of Knowledge: GEOMETRY

Standard 1: Points, Lines, Angles, and Planes

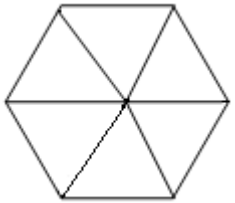
Understand geometric concepts, applications, and their representations with coordinate systems. Find lengths and midpoints of line segments, slopes, parallel and perpendicular lines, and equations of lines. Using a compass and straightedge, patty paper, a drawing program or other techniques, construct lines and angles, explaining and justifying the processes used.

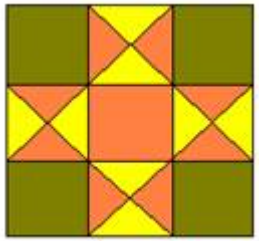
BENCHMARK CODE	BENCHMARK
MA.912.G.1.1	Find the lengths and midpoints of line segments in two-dimensional coordinate systems. *Remarks/Examples*: Example: Find the length and midpoint of the line segment joining the points A (3, -8) and B (9, 0). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.1.2	Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass or a drawing program, explaining and justifying the process used. *Remarks/Examples*: Example 1: Draw a triangle ABC. Duplicate it using your compass and straightedge. Example 2: Construct the perpendicular bisector of a given line segment, justifying each step of the process. Example 3: The city shown below has two offices (A and B) for the same newspaper. The two offices want to divide the city into two regions so that any location in one region is always closer to their own newspaper office than to the other one. Copy the map and locate the dividing line. Explain why this line meets the given criteria. Select several points in each region and make sure they are closer to their newspaper office than they are to the other one.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.1.3	Identify and use the relationships between special pairs of angles formed by parallel lines and transversals. *Remarks/Examples*: Example: In the diagram, the lines k and l are parallel. Find the value of x. Find all angle values in the diagram. Explain your answer.

	 Example 2: In the diagram, the lines m and n are parallel. Find the value of x. Explain your answer.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.1.4	Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines. *Remarks/Examples*: Example 1: Given points P(2,-1), Q(-4, 2), and M(5,3), find the coordinates of a point N such that $\overleftrightarrow{PQ}$ and $\overleftrightarrow{MN}$ are parallel. Find coordinates of a point K such that $\overleftrightarrow{MK}$ is perpendicular to $\overleftrightarrow{PQ}$. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 2: Polygons

Identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.), using terms such as regular, convex, and concave. Find measures of angles, sides, perimeters, and areas of polygons, justifying the methods used. Apply transformations to polygons. Relate geometry to algebra by using coordinate geometry to determine transformations. Use algebraic reasoning to determine congruence, similarity, and symmetry. Create and verify tessellations of the plane using polygons.

BENCHMARK CODE	BENCHMARK
MA.912.G.2.1	Identify and describe convex, concave, regular, and irregular polygons. *Remarks/Examples*: Example 1: Draw a hexagon. Is it convex or concave? Is it regular or irregular? Explain your answers. Example 2: Define the terms convex, concave, regular and irregular polygon and draw a picture of the tern next to the definition. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.2.2	Determine the measures of interior and exterior angles of polygons, justifying the method used. *Remarks/Examples*: Example 1: Calculate the measure of one interior angle and one exterior of a regular octagon. Explain your method. Example 2: Suppose that you will make a picture frame like the one shown below. To make the regular hexagonal frame, you will use identical trapezoidal pieces. What are the measures of the angles of the trapezoids? Explain your answer.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.2.3	Use properties of congruent and similar polygons to solve mathematical or real-world problems. *Remarks/Examples*: Example: Suppose a building is in the shape of a regular hexagon. The architect wants to put walkways as indicated. Show that the triangles formed are equal in size and shape.  *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.2.4	Apply transformations (translations, reflections, rotations, dilations, and scale factors) to polygons. to determine congruence, similarity, and symmetry. Know that images formed by translations, reflections, and rotations are congruent to the original shape. Create and verify tessellations of the plane using polygons. *Remarks/Examples*: Physical objects, drawings, and dynamic geometry software might help students explore this benchmark. Students' early work in

	elementary and middle school should form a base for teaching this benchmark (see MA.3.G.3.3, MA.4.G.5.2, and MA.7.G.4.2). Students should explore different types of transformations and observe that some transformations (translations, reflections, and rotations) result in congruent shapes. Example: Explore regular polygons through manipulatives and/or drawing programs. Describe which of the polygons would be best for tiling a rectangular floor. Explain your reasoning. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.2.5	Explain the derivation and apply formulas for perimeter and area of polygons (triangles, quadrilaterals, pentagons, etc.). *Remarks/Examples*: Example 1: A rectangle of area 360 square yards is ten times as long as it is wide. Find its length and width. Example 2: Explain the derivation of the formula for the area of a triangle. Example 3: The design below is called the Ohio Star. Assuming that it measures 9 inches by 9 inches, calculate the total area of all the orange patches, the total area of all the yellow patches, and the total area of all the green patches. How much fabric of each color will you need to cover an area that measures 72 inches by 90 inches?  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.2.6	Use coordinate geometry to prove properties of congruent, regular and similar polygons, and to perform transformations in the plane. *Remarks/Examples*: Example: Draw the polygon defined by the following vertices (1, 3), (-1, 3), (3, 1), (-3, 1), (1, -3), (-1, -3), (-3, -1), (3, -1). Is this polygon regular? Justify your answer. Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6), and (1, 5) a square? Justify your answer. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.2.7	Determine how changes in dimensions affect the perimeter and area of common geometric figures. *Remarks/Examples*:

	Example: If the lengths of each side of a trapezoid are tripled, determine the change in its area, and justify your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

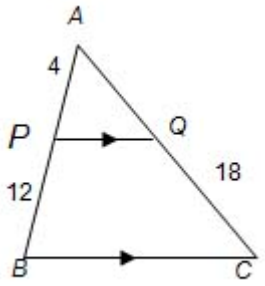
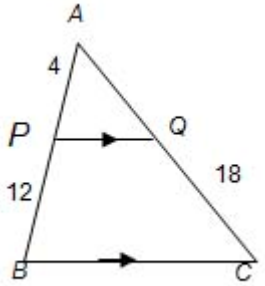
Standard 3: Quadrilaterals Classify and understand relationships among quadrilaterals (rectangle, parallelogram, kite, etc.). Relate geometry to algebra by using coordinate geometry to determine regularity, congruence, and similarity. Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas, and prove theorems involving quadrilaterals.	
BENCHMARK CODE	**BENCHMARK**
MA.912.G.3.1	Describe, classify, and compare relationships among quadrilaterals including the square, rectangle, rhombus, parallelogram, trapezoid, and kite. *Remarks/Examples*: This benchmark examines properties of quadrilaterals one at a time. Example: Explore a trapezoid through manipulatives, drawings and/or technology. Draw the diagonals and determine whether they are perpendicular. Give a convincing argument that your judgment is correct. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.3.2	Compare and contrast special quadrilaterals on the basis of their properties. *Remarks/Examples*: This benchmark examines similarities and differences between different types of quadrilaterals. Example: Explain the similarities and differences between a rectangle, rhombus, and kite. Create a Venn diagram to match your explanation. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.3.3	Use coordinate geometry to prove properties of congruent, regular, and similar quadrilaterals. *Remarks/Examples*: Coordinate geometry is used while students prove quadrilaterals to be congruent, similar, or regular. Coordinate geometry is used to prove properties of quadrilaterals. Example: Given a quadrilateral with vertices (0, 0), (5/2, 5sqrt(3)/2), (5, 0), (7, 7sqrt(3)/3), prove that the diagonals of this quadrilateral are perpendicular.

	Example: Is rectangle *ABCD* with vertices at A(0, 0), B(4, 0), C(4, 2), D(0, 2) congruent to rectangle *PQRS* with vertices at P(-2, -1), Q(2, -1), R(2, 1), S(-2, 1)? Justify your answer. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.3.4	Prove theorems involving quadrilaterals. *Remarks/Examples*: Example: Prove that the diagonals of a rectangle are congruent. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 4: Triangles

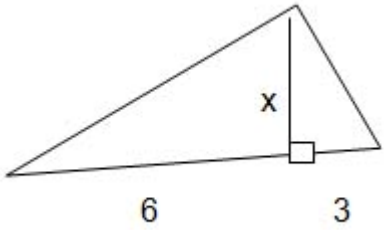
Identify and describe various kinds of triangles (right, acute, scalene, isosceles, etc.). Define and construct altitudes, medians, and bisectors, and triangles congruent to given triangles. Prove that triangles are congruent or similar and use properties of these triangles to solve problems involving lengths and areas. Relate geometry to algebra by using coordinate geometry to determine regularity, congruence, and similarity. Understand and apply the inequality theorems of triangles.

BENCHMARK CODE	BENCHMARK
MA.912.G.4.1	Classify, construct, and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular. *Remarks/Examples*: Students may use a compass and straightedge or a drawing program to construct and classify triangles, and describe the attributes of each triangle. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.4.2	Define, identify, and construct altitudes, medians, angle bisectors, perpendicular bisectors,orthocenter, centroid, incenter, and circumcenter. *Remarks/Examples*: Example: Draw several triangles. Construct their angle bisectors. What do you observe from your drawings? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.4.3	Construct triangles congruent to given triangles. *Remarks/Examples*: Example: Given a triangle, construct a congruent triangle and prove that the two triangles are congruent. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.4.4	Use properties of congruent and similar triangles to solve problems involving lengths and areas. *Remarks/Examples*: Example: Of two similar triangles, the second has sides half the length of the first. The area of the first triangle is 20 cm^2. What is the area of the second triangle? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.4.5	Apply theorems involving segments divided proportionally.

	Remarks/Examples: Example: In triangle ABC shown below, $\overline{PQ}$ is parallel to $\overline{BC}$. What is the length of $\overline{AQ}$?  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.4.6	Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles. *Remarks/Examples*: Example: Prove that triangles ABC and APQ are similar.  *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.4.7	Apply the inequality theorems: triangle inequality, inequality in one triangle, and the Hinge Theorem. *Remarks/Examples*: Example: Can you draw a triangle with sides of length 7 cm, 4 cm, and 15 cm? Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.4.8	Use coordinate geometry to prove properties of congruent, regular, and similar triangles. *Remarks/Examples*: Example: Draw a triangle with vertices at (1, 3), (2, 5), and (6, 1). Draw another triangle with vertices at (-3, -1), (-2, 1), and (2, -3). Are these triangles congruent, similar or neither? Defend your answer. *Cognitive Complexity/Depth of Knowledge Rating:* High

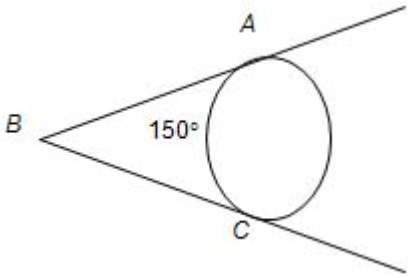
Standard 5: Right Triangles

Apply the Pythagorean Theorem to solving problems, including those involving the altitudes of right triangles and triangles with special angle relationships. Use special right triangles to solve problems using the properties of triangles.

BENCHMARK CODE	BENCHMARK
MA.912.G.5.1	Prove and apply the Pythagorean Theorem and its converse. *Remarks/Examples*: Example: Determine if the triangle with side lengths of 10, 12, and 18 is a right triangle. Justify your reasoning. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.5.2	State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. *Remarks/Examples*: Example: Find the value of x in the right triangle below.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.5.3	Use special right triangles (30° - 60° - 90° and 45° - 45° - 90°) to solve problems. *Remarks/Examples*: Example: An isosceles right triangle has one leg 6 cm long. Find the lengths of the other two sides. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.5.4	Solve real-world problems involving right triangles. *Remarks/Examples*: Example: The distance of the base of a ladder from the wall it leans against should be at least 1/3 of the ladder's total length. Suppose a 12-ft ladder is placed according to these guidelines. Give the minimum distance of the base of the ladder from the wall. How far up the wall will the ladder reach? Explain and include a sketch in your explanation. *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 6: Circles

Define and understand ideas related to circles (radius, tangent, chord, etc.). Perform constructions, and prove theorems related to circles. Find measures of arcs and angles related to them, as well as measures of circumference and area. Relate geometry to algebra by finding the equation of

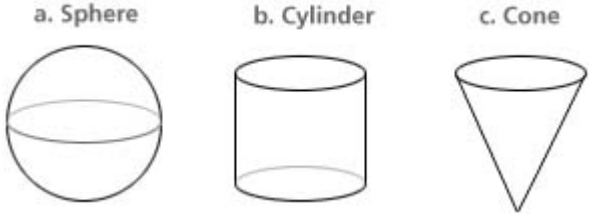
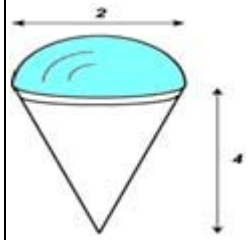
a circle in the coordinate plane.	
BENCHMARK CODE	**BENCHMARK**
MA.912.G.6.1	Determine the center of a given circle. Given three points not on a line, construct the circle that passes through them. Construct tangents to circles. Circumscribe and inscribe circles about and within triangles and regular polygons. *Remarks/Examples*: Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords. Example: Given a circle and a point on the circle, construct a tangent to the circle, passing through the given point. Example: Draw an acute triangle and construct the circumscribed circle. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.6.2	Define and identify: circumference, radius, diameter, arc, arc length, chord, secant, tangent and concentric circles. *Remarks/Examples*: Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle? *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.G.6.3	Prove theorems related to circles, including related angles, chords, tangents, and secants. *Remarks/Examples*: Example: Prove that a segment from the center of a circle perpendicular to a chord, bisects the chord. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.6.4	Determine and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents). *Remarks/Examples*: Example: Find the measure of angle ABC in the diagram below.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.6.5	Solve real-world problems using measures of circumference, arc length, and areas of circles and sectors. *Remarks/Examples*: Example: Which will give you more: three 6-inch pizzas or two 8-inch pizzas? Explain your answer. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.6.6	Given the center and the radius, find the equation of a circle in the coordinate plane or given the equation of a circle in center-radius

	form, state the center and the radius of the circle. *Remarks/Examples*: Example: Find the equation of the circle with radius 10 and center (6, -3). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.6.7	Given the equation of a circle in center-radius form or given the center and the radius of a circle, sketch the graph of the circle. *Remarks/Examples*: Example: Sketch the graph of the circle whose equation is $(x-3)^2+(y+2)^2=16$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 7: Polyhedra and Other Solids

Describe and make regular and nonregular polyhedra (cube, pyramid, tetrahedron, octahedron, etc.). Explore relationships among the faces, edges, and vertices of polyhedra. Describe sets of points on spheres, using terms such as great circle. Describe symmetries of solids, and understand the properties of congruent and similar solids.

BENCHMARK CODE	BENCHMARK
MA.912.G.7.1	Describe and make regular, non-regular, and oblique polyhedra, and sketch the net for a given polyhedron and vice versa. *Remarks/Examples*: Example: Make a net for a tetrahedron out of poster board and fold it up to make the tetrahedron. Is this a regular polyhedron? Explain why or why not. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.7.2	Describe the relationships between the faces, edges, and vertices of polyhedra. *Remarks/Examples*: Example: Use manipulatives to investigate the relationships between faces, edges, and vertices of polyhedra; i.e., Euler's Theorem. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.7.3	Identify, sketch, find areas and/or perimeters of cross sections of solid objects. *Remarks/Examples*: Example: What cross sections can you get from each of these figures? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.7.4	Identify chords, tangents, radii, and great circles of spheres *Remarks/Examples*: Example: On Earth, is the equator a great circle? Explain your answer.

	 *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.G.7.5	Explain and use formulas for lateral area, surface area, and volume of solids. *Remarks/Examples*: Example: A gold class ring is dropped into a glass that is a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the ring? Example: Given the composite solid consisting of a hemisphere and a cone, calculate the surface area and the volume.  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.7.6	Identify and use properties of congruent and similar solids. *Remarks/Examples*: Example: Explain how the surface area and volume of similar cylinders are related *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.7.7	Determine how changes in dimensions affect the surface area and volume of common geometric solids. *Remarks/Examples*: Example: Explain how changing the radius or height of a cylinder affects its surface area and volume. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 8: Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, use problem-solving skills, choose how to approach a problem, explain the reasoning, and check the results. At this level, apply these skills to making conjectures, using axioms and theorems, constructing logical arguments, and writing geometric proofs. Learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false.

BENCHMARK CODE	BENCHMARK
MA.912.G.8.1	Analyze the structure of Euclidean geometry as an axiomatic system. Distinguish between undefined terms, definitions, postulates, and theorems.

	Remarks/Examples: Example: Classify each of the following as an undefined term, defined term, postulate, or theorem: • Line • Isosceles triangle • Regular hexagon • Pythagorean Therom Students should also explore non-Euclidean geometries including hyperbolic and elliptic geometries. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.8.2	Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards. *Remarks/Examples*: Example: How far does the tip of the minute hand of a clock move in 20 minutes if the tip is 4 inches from the center of the clock? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.8.3	Determine whether a solution is reasonable in the context of the original situation. *Remarks/Examples*: Example: The area of a circle is 49p and George determined that the diameter is 7. Is his answer reasonable? Why or why not? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.G.8.4	Make conjectures with justifications about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture. *Remarks/Examples*: Example: Calculate the ratios of side lengths in several different-sized triangles with angles of 90°, 50°, and 40°. What do you notice about the ratios? How might you prove that your observation is true (or show that it is false)? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.8.5	Write geometric proofs, including proofs by contradiction and proofs involving coordinate geometry. Use and compare a variety of ways to present deductive proofs, such as flow charts, paragraphs, two-column, and indirect proofs. *Remarks/Examples*: Example: Prove that the sum of the measures of the interior angles of a triangle is 180°. Example: Prove that the perpendicular bisector of line segment AB is the set of all points equidistant from the endpoints A and B. Example: Prove that two lines are parallel if and only if the alternate interior angles the lines make with a transversal are equal. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.G.8.6	Perform basic constructions using straightedge and compass, and/or drawing programs describing and justifying the procedures used. Distinguish between sketching, constructing, and drawing geometric figures. *Remarks/Examples*: Example: Construct a line parallel to a given line through a given point not on the line, explaining and justifying each step.

	Cognitive Complexity/Depth of Knowledge Rating: High

Body of Knowledge: PROBABILITY

Standard 1: Counting Principles

Understand the counting principle, permutations, and combinations, and use them to solve problems.

BENCHMARK CODE	BENCHMARK
MA.912.P.1.1	Use counting principles, including the addition and the multiplication principles, to determine size of finite sample spaces and probabilities of events in those spaces. *Remarks/Examples*: Example: A dinner menu has three choices for appetizers, five choices for main dishes, and four options for dessert. How many different choices of one appetizer, one main dish, and one dessert are there? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.P.1.2	Use formulas for permutations and combinations to count outcomes and determine probabilities of events. *Remarks/Examples*: Example: You are one of 15 potential members of a committee. A committee of 4 people will be chosen at random from the 15. How many possible committees can be formed? If 6 of the potential members are women, what is the probability of all the committee members being women? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 2: Determine Probabilities

Develop rules for finding probabilities of combined and complementary events. Understand and use conditional probability and the related Bayes' Theorem.

BENCHMARK CODE	BENCHMARK
MA.912.P.2.1	Determine probabilities of complementary events, and calculate odds for and against the occurrence of events. *Remarks/Examples*: Example: Suppose Antonio makes 75% of his foul shots in basketball. If he gets to attempt two shots, what is the probability of his making at least one of the two shots? What are the odds against missing both shots? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.P.2.2	Determine probabilities of independent events. *Remarks/Examples*: Example: A fair coin is tossed four times. What is the probability of getting heads on at least two of the tosses? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.P.2.3	Understand and use the concept of conditional probability, including: understanding how conditioning affects the probability of events and finding conditional probabilities from a two-way frequency table. *Remarks/Examples*:

	Example: In a certain large city, 25% of all wage earners have a college degree. Of those who do have a college degree, 10% earn more than $80,000 per year, and of those who do not, 4% earn more than $80,000 per year. If a randomly selected wage earner earns more than $80,000 per year, what is the probability that (s)he has a college degree? *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 3: Probability Distributions

Investigate probability distributions, and calculate and interpret their means and variances. Use and apply the normal distribution, including using the central limit theorem.

BENCHMARK CODE	BENCHMARK
MA.912.P.3.1	Determine probabilities of events from distributions, including: • discrete uniform (all outcomes in a finite set equally likely) • binomial • normal • exponential *Remarks/Examples*: Example: Suppose that 60% of the general population are basketball fans. If 8 people are chosen at random, what is the probability that 4 of them will be basketball fans? Example: Math SAT scores are normally distributed with mean 500, standard deviation 100. What is the probability that Joan's SAT score is greater than 550? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.P.3.2	Determine the mean and variance of distributions, including: • discrete uniform (all outcomes in a finite set equally likely) • binomial • normal • exponential *Remarks/Examples*: Example: A fair coin is flipped 10 times. Find the mean and variance of the number of heads. Example: continuous distribution *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.P.3.3	Apply the properties of the normal distribution. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.P.3.4	Apply the Central Limit Theorem to determine the probability that a sample mean will be in a certain interval. *Remarks/Examples*: Example: During a certain week the mean price of gasoline in Florida was $2.164 per gallon. What is the probability that the mean price for a sample of 38 gas stations in Florida is between $2.169 and $2.179? Assume the population standard deviation = $0.049.

	Cognitive Complexity/Depth of Knowledge Rating: High

Body of Knowledge: STATISTICS

Standard 1: Formulating Questions

Learn to define appropriate questions for research and to pose questions in a form that can be answered by collecting and analyzing data.

BENCHMARK CODE	BENCHMARK
MA.912.S.1.1	Formulate an appropriate research question to be answered by collecting data or performing an experiment. *Remarks/Examples*: Example: An article in the local paper states that the health of Americans has declined over the past decade. How can this assertion be stated in a way that allows for scientific testing? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.1.2	Determine appropriate and consistent standards of measurement for the data to be collected in a survey or experiment. *Remarks/Examples*: Example: A student is designing a survey to gauge levels of stress in a population of high schools students. Is "stress" something that can be directly measured? How should the student define "stress" so that it can be objectively and consistently measured? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 2: Data Collection

Learn key methods for collecting data and basic sampling principles.

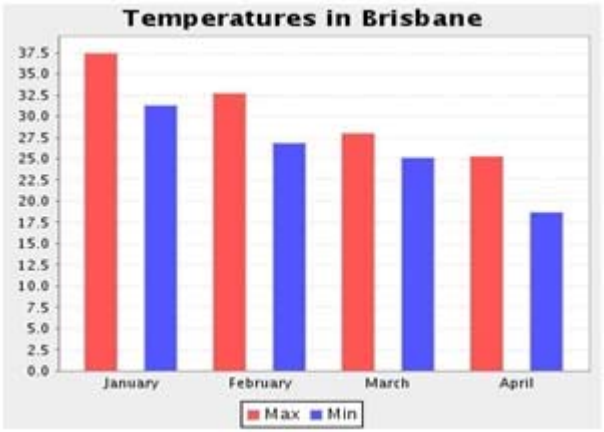
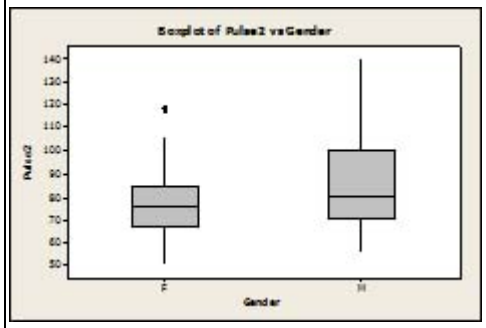
BENCHMARK CODE	BENCHMARK
MA.912.S.2.1	Compare the difference between surveys, experiments, and observational studies and what types of questions can and cannot be answered by a particular design. *Remarks/Examples*: Example: Which kind of statistical study should be used (and why) to study each of the following: a) What percent of the voting age population in Florida favors making English the official language? b) What wavelength of light is best for plant growth? c) What is the relationship between x-ray exposure and cancer rates? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.2.2	Apply the definition of random sample and basic types of sampling, including representative samples, stratified samples, censuses. *Remarks/Examples*: Example: A survey is being planned to determine public opinion on a pending proposal to reform Social Security, and specifically whether there are differences by age group, political affiliation, and educational level. Choose and justify the appropriate type of sample to use? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.2.3	Identify sources of bias, including sampling and nonsampling errors.

	Remarks/Examples: Example: An Internet poll is conducted to determine the average educational level of adults in Florida. Describe possible sources of bias in the results of the poll. How useful are the results? Example: A survey asks a sample of students questions about their drug use. What sources of bias might enter into the results? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

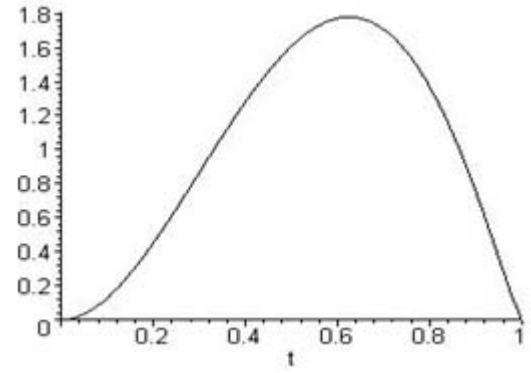
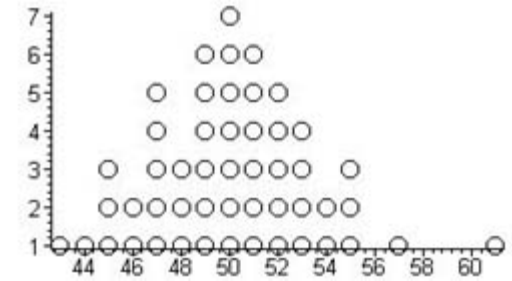
Standard 3: Summarizing Data (Descriptive Statistics)

Learn to work with summary measures of sets of data, including measures of the center, spread, and strength of relationship between variables. Learn to distinguish between different types of data and to select the appropriate visual form to present different types of data.

BENCHMARK CODE	BENCHMARK
MA.912.S.3.1	Read and interpret data presented in various formats. Determine whether data is presented in appropriate format, and identify possible corrections. Formats to include: • bar graphs • line graphs • stem and leaf plots • circle graphs • histograms • box and whiskers plots • scatter plots • cumulative frequency (ogive) graphs *Remarks/Examples*: Example: The chart below shows the average daily high and low temperatures in an Australian city. What is the average high temperature in January? What is the average low temperature in March? Which month has higher temperatures, January or April? Can you think of a reason for this? Example: Compare the distributions of pulse rates for males and females by interpreting the following box and whisker plots:

	  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.3.2	Collect, organize, and analyze data sets, determine the best format for the data and present visual summaries from the following: • bar graphs • line graphs • stem and leaf plots • circle graphs • histograms • box and whisker plots • scatter plots • cumulative frequency (ogive) graphs *Remarks/Examples*: Example: Gather data to answer the question: which area of the country has the highest dropout rate? Display your dropout data in appropriate formats. Example: given a set of data, use appropriate technology to sort the data and to display a histogram or other chart. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.3.3	Calculate and interpret measures of the center of a set of data, including mean, median, and weighted mean, and use these measures

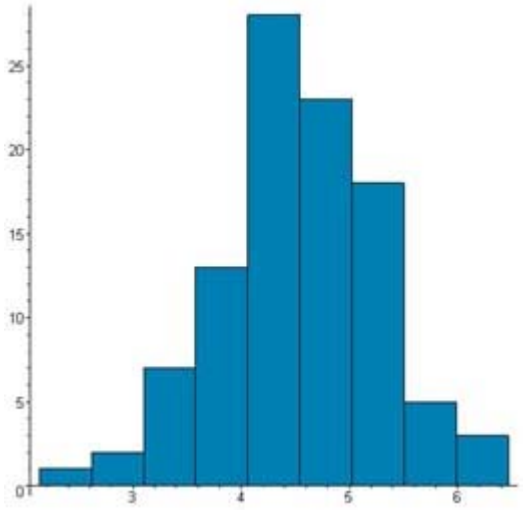
	to make comparisons among sets of data. *Remarks/Examples*: Example: A sample of five runs for bus A had passenger loads of 15, 24, 19, 12, and 20 passengers. A similar sample for bus B had passenger loads of 18, 21, 16, 14, and 16 passengers. Based on these samples, calculate the mean and median for the number of passengers for each bus. Which bus carries larger passenger loads? How does the answer to that question depend on which measure is being used (mean verses median)? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.3.4	Calculate and interpret measures of variance and standard deviation. Use these measures to make comparisons among sets of data. *Remarks/Examples*: Example: Monthly average high temperatures for Orlando are: 72, 73, 77, 83, 88, 91, 92, 92, 89, 84, 77, 73, while monthly average high temperatures for Tallahassee are: 64, 67, 73, 80, 87, 90, 91, 91, 88, 81, 72, 65. Which city has the greater variation in average high temperatures? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.3.5	Calculate and interpret the range and quartiles of a set of data. *Remarks/Examples*: Example: Scores on a recent math test in a certain class were as follows: 77, 84, 91, 50, 75, 95, 62, 83, 85, 78, 68, 92, 74, 81, 92, 98, 83, 73, 100, 71. Find the range of the test scores, and compute the interquartile range (IQR). *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.3.6	Use empirical rules such as the 68-95-99.7 rule to estimate spread of distributions and to make comparisons among sets of data. *Remarks/Examples*: Example: The weights, in grams, of 16 randomly selected mice are: 15.7, 13.1, 13.9, 13.4, 14.8, 16.9, 14.2, 14.7, 13.7, 15.8, 16.7, 15.6, 16.1, 16.3, 14.1, 17.0. Find the variance of this set of data, and use the empirical rule to estimate the range of weights of the entire population of mice. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.3.7	Calculate the correlation coefficient of a set of paired data, and interpret the coefficient as a measure of the strength and direction of the relationship between the variables. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.3.8	Determine whether a data distribution is symmetric or skewed based on an appropriate graphical presentation of the data. *Remarks/Examples*: Example: The graph below shows the probability density function of a continuous distribution. Determine whether the distribution is skewed left, skewed right, or symmetric.

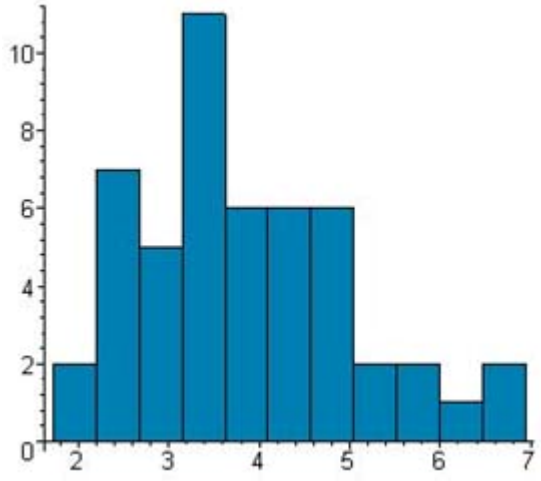
	 *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.S.3.9	Identify outliers in a set of data based on an appropriate graphical presentation of the data, and describe the effect of outliers on the mean, median, and range of the data. *Remarks/Examples*: Example: The dotplot for the number of hours worked by 50 employees in one week at a firm is shown below. Are there any points that appear to be outliers?  *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 4: Analyzing Data

Learn to use simulations of standard sampling distributions to determine confidence levels and margins of error. Develop measures of association between two numerical or categorical variables. Use technological tools to find equations of regression lines and correlation coefficients.

BENCHMARK CODE	BENCHMARK
MA.912.S.4.1	Explain and interpret the concepts of confidence level and "margin of error." *Remarks/Examples*: Example: A newspaper article states that a recent poll on a topic has a margin of error of plus or minus 4%. Explain what this means in terms of the distribution of the actual population. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.4.2	Use a simulation to approximate sampling distributions for the mean, using repeated sampling simulations from a given population.

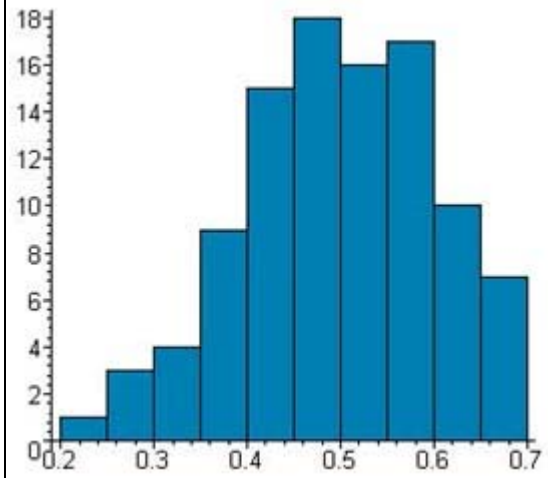
	Remarks/Examples: Example: The histogram below shows the distribution of the sample mean when samples of 15 random digits are selected (with replacement) and the sample mean is computed, with the process repeated 100 times to generate 100 sample means. What type of distribution does this resemble? If you visually estimate the mean from the histogram, how does it compare with the mean of the original uniform distribution?  *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.4.3	Apply the Central Limit Theorem to solve problems. *Remarks/Examples*: Example: Suppose you plotted a histogram of 100 sample means from a certain distribution. Would the histogram be more symmetrical if you used samples of size 10 or of size 20? Which sample size would result in a more widely spread out histogram? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.4.4	Approximate confidence intervals for means using simulations of the distribution of the sample mean. *Remarks/Examples*: Example: In an observational study, the waiting times for service of 10 randomly chosen customers in one day at a fast food restaurant are recorded, and the mean of the sample noted. This process is repeated for 50 days, with the resulting histogram of the sample means shown below. Use this histogram to estimate a 90% confidence interval for the mean wait time at this restaurant.

	 *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.4.5	Find the equation of the least squares regression line for a set of data. *Cognitive Complexity/Depth of Knowledge Rating:* Low

Standard 5: Interpreting Results

Gather data and determine confidence intervals to make inferences about means, and use hypothesis tests to make decisions. Learn to use data to approximate p-values and to determine whether correlations between variables are significant.

BENCHMARK CODE	BENCHMARK
MA.912.S.5.1	Analyze the relationship between confidence level, margin of error, and sample size. *Remarks/Examples*: Example: A pollster wishes to estimate the proportion of United States voters who favor capital punishment. How large a sample is needed in order to be 95% confident that the sample proportion will not differ from the true proportion by more than 2%? Example: Compared to a margin of error based on 95% confidence and a sample size of n=36, explain how margin of error changes when A) 99% confidence is used. B) A sample size of n=30 is used. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.5.2	Apply the general principles of hypothesis testing. *Remarks/Examples*: Example: Can you use a hypothesis test to prove that the average height of an adult male is 6 feet? Why or why not? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.5.3	Explain and identify the following: null hypothesis, alternative hypotheses, Type I error, and Type II error. *Remarks/Examples*:

	Example: According to the norms established for a history test, eighth graders should average 81.7 with a standard deviation of 8.5. a. Identify null and alternative hypotheses to be used for an experiment to test students' performance on the test. b. Explain what a Type I and a Type II error would be in the context of the null and alternative hypotheses given in (a). *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.5.4	Explain the meaning of p-value and its role in hypothesis testing. *Remarks/Examples*: Example: A statistical analysis of an experiment yields a p-value of 0.02. Explain the meaning of this p-value in terms of the decision that is made about the null and alternative hypotheses and Type I error. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.5.5	Perform hypothesis tests of means and proportions for large samples, using simulations to determine whether a sample mean (proportion) has a low likelihood of occurring. *Remarks/Examples*: Example: A student wants to determine whether a certain coin is fair. She flips it 20 times, and notes that it came up heads 65% of the time (13 times out of 20). A computer simulation of the same experiment with a fair coin, repeated 100 times, yielded varying results, shown in the histogram below. How many of the 100 experiments done by the computer resulted in 65% or more heads?  *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.5.6	Interpret the results of hypothesis tests of means and proportions, and make decisions based on p-values of test. *Remarks/Examples*: Example: In an effort to determine whether a school's ACT scores are going up, the school looked at a sample of scores from 10 current seniors, and found the sample mean to be 27.5. Historically, scores have been normally distributed with a mean of 25 and standard deviation of 4. If the school adopts a null hypothesis that the mean is still 25, and a one-sided alternative, the sample mean yields a p-value of 0.023. Determine whether this is good evidence that ACT scores have gone up, and write a summary explanation of your decision suitable for a presentation to a non-statistics-minded audience. If the alternative hypothesis were two-sided, what would the p-value be? *Cognitive Complexity/Depth of Knowledge Rating:* High

MA.912.S.5.7	Use simulations to approximate the p-value of a correlation coefficient, and use the results to determine whether the correlation between two variables is significant. *Remarks/Examples*: This benchmark includes having students recognize when arguments based on data confuse correlation with causation. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.S.5.8	Use a regression line equation to make predictions. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.S.5.9	Interpret the coefficient of determination, r^2, for a least-squares regression. *Remarks/Examples*: Example: A study found the correlation between the weight of a vehicle (in pounds) and its gas mileage (MPG) to be r = - 0.65. What percent of the variation in MPG can be explained by the straight-line relationship with vehicle weight? *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Body of Knowledge: TRIGONOMETRY

Standard 1: Trigonometric Functions

Extend the definitions of the trigonometric functions beyond right triangles using the unit circle, and measure angles in radians as well as degrees. Draw and analyze graphs of trigonometric functions (including finding period, amplitude, and phase shift), and use them to solve word problems. Define and graph inverse trigonometric functions, and determine values of both trigonometric and inverse trigonometric functions.

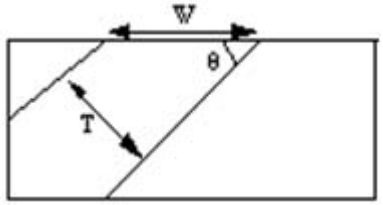
BENCHMARK CODE	BENCHMARK
MA.912.T.1.1	Convert between degree and radian measures. *Remarks/Examples*: Example: Convert $90^\circ, 45^\circ, 30^\circ$ to radians. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.1.2	Define and determine sine and cosine using the unit circle. *Remarks/Examples*: Example: Find the acute angle, θ, for which $sin(150^\circ) = sin(\theta)$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.1.3	State and use exact values of trigonometric functions for special angles: multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (degree and radian measures). *Remarks/Examples*: Example: State the exact values of $\cos\left(\frac{\pi}{2}\right)$ $\tan\left(\frac{3\pi}{4}\right)$ $\csc\left(\frac{2\pi}{3}\right)$ $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

	Cognitive Complexity/Depth of Knowledge Rating: Low
MA.912.T.1.4	Find approximate values of trigonometric and inverse trigonometric functions using appropriate technology. *Remarks/Examples*: Example: Find the approximate values for $tan^{-1}(1.73)$and $sin(55°)$ *Cognitive Complexity/Depth of Knowledge Rating:* Low
MA.912.T.1.5	Make connections between right triangle ratios, trigonometric functions, and circular functions. *Remarks/Examples*: Example: Angle $\theta$is a 50º angle of a right triangle with a hypotenuse of length 14. Find the exact value for sine, cosine, and tangent of angle θ. Example: Find the real numbers x, 0 < x < 2p, with exactly the same sine value as θ Example: Find the real numbers x, 0 < x < 2p, with exactly the same sine value as *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.1.6	Define and graph trigonometric functions using domain, range, intercepts, period, amplitude, phase shift, vertical shift, and asymptotes with and without the use of graphing technology. *Remarks/Examples*: Example: Graph y=sin x and y=cos x and compare their graphs. Example: Find the asymptotes of y=tan xand find its domain. Example: Draw the graph of $y - 5 + sin(2x - \frac{\pi}{3})$ *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.T.1.7	Define and graph inverse trigonometric relations and functions. *Remarks/Examples*: Example: Graph $f(x) = sin^{-1}x$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.1.8	Solve real-world problems involving applications of trigonometric functions using graphing technology when appropriate. *Remarks/Examples*: Example: The number of hours of daylight varies through the year in any location. A graph of the number of hours of daylight throughout the year is in the form of a sine wave. In a certain location the longest day of 14 hours is on Day 175 and the shortest day of 10 hours is on Day 355. Sketch a graph of this function and find its equation. Which other day has the same length as July 4 (Day 186)? *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 2: Trigonometry in Triangles

Understand how the trigonometric functions relate to right triangles, and solve word problems involving right and oblique triangles. Understand and apply the laws of sines and cosines. Use trigonometry to find the area of triangles.	
BENCHMARK CODE	**BENCHMARK**
MA.912.T.2.1	Define and use the trigonometric ratios (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles. *Remarks/Examples*: Example: In triangle ABC, tan A = 1/5. Find sin A and cot A. Example: Show that the slope of a line at 135º to the x-axis is the same as the tangent of 135º. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.2.2	Solve real-world problems involving right triangles using technology when appropriate. *Remarks/Examples*: Example: The elevation of the Pensacola Lighthouse in Pensacola, Florida is 191 feet above sea level. From the top of the light house, the angle of depression to a fishing boat in the Gulf of Mexico is determined to be 15o. How far is the fishing boat from the lighthouse? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.T.2.3	Apply the laws of sines and cosines to solve real-world problems using technology. *Remarks/Examples*: Example: You want to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78º. From the second position, the angle between the mountain and the first position is 53º. How far is the mountain from each position? *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.T.2.4	Use the area of triangles given two sides and an angle or three sides to solve real-world problems. *Remarks/Examples*: Example: Calculate the surface area of carpet you need to purchase (in square meters) to cover the floor of a triangle-shaped file cabinet room with sides of length 8 m and 6 m enclosing an angle of 60º. Example: Use Heron's formula to find the area of a triangle with side lengths 4, 7, and 9. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 3: Trigonometric Identities and Equations Know basic trigonometric identities derived from definitions, and use them to prove other identities. Use the sum, difference, double-angle, and half-angle formulas. Solve trigonometric equations and word problems using trigonometry.	
BENCHMARK CODE	**BENCHMARK**
MA.912.T.3.1	Verify the basic Pythagorean identities, such as $sin^2x + cos^2x = 1$, and show they are equivalent to the Pythagorean Theorem. *Remarks/Examples*: Example: Use a right triangle to show that $sin^2x + cos^2x = 1$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.3.2	Use basic trigonometric identities to verify other identities and simplify expressions. *Remarks/Examples*:

	Example: Show that $\frac{tan^2x}{1+tan^2x} = sin^2x$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.3.3	Use the sum and difference, half-angle and double-angle formulas for sine, cosine, and tangent, when formulas are provided. *Remarks/Examples*: Example: Find and verify the exact value of the $cos(75^\circ)$ using two different trigonometric identities. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.3.4	Solve trigonometric equations and real-world problems involving applications of trigonometric equations using technology when appropriate. *Remarks/Examples*: Example: The diagram above shows a vertical cross-section through some strata, showing that the width (W) of the middle layer, as measured on the upper surface, is not the true width of the layer, T, which is assumed to be constant. Calculate the true width, T, when theta=30º, W=100 meters. Example: Solve 2 sin(x) +1=0 on the interval [0, 2¶)  *Cognitive Complexity/Depth of Knowledge Rating:* High

Standard 4: Polar Coordinates and Trigonometric Form of Complex Numbers

Define, use polar coordinates, and relate them to Cartesian coordinates. Translate equations in terms of Cartesian coordinates into polar coordinates, and graph the resulting equations in the polar coordinate plane. Convert complex numbers from standard to trigonometric form, and vice-versa. Multiply complex numbers in trigonometric form, and use De Moivre's Theorem.

BENCHMARK CODE	BENCHMARK
MA.912.T.4.1	Define polar coordinates and relate polar coordinates to Cartesian coordinates with and without the use of technology. *Remarks/Examples*: Example: Convert the polar coordinates $(2, \pi/3)$ to (x, y) form. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.4.2	Represent equations given in rectangular coordinates in terms of polar coordinates. *Remarks/Examples*: Example: Represent the equation $x^2 + y^2 = 4$ in terms of polar coordinates. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

MA.912.T.4.3	Graph equations in the polar coordinate plane with and without the use of graphing technology. *Remarks/Examples*: Example: Graph $y = 1 - cos\theta$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.4.4	Define the trigonometric form of complex numbers, convert complex numbers to trigonometric form, and multiply complex numbers in trigonometric form. *Remarks/Examples*: Example: Write 3 + 3i and 2 - 4i in trigonometric form; multiply the results; and write the answer in a + bi form. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.4.5	Apply DeMoivre's Theorem to perform operations with complex numbers. *Remarks/Examples*: Example: Simplify $(1 - i)^{23}$ *Cognitive Complexity/Depth of Knowledge Rating:* Moderate

Standard 5: Mathematical Reasoning and Problem Solving

Use a variety of strategies to solve problems. Develop and evaluate mathematical arguments and proofs.

BENCHMARK CODE	BENCHMARK
MA.912.T.5.1	Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, examining simpler problems, and working backwards, using technology when appropriate. *Remarks/Examples*: Example: Graph y = sin x + cos x without the use of graphing technology. Students should work problems where they are required to distinguish relevant from irrelevant information, identify missing information, and either find missing data or make appropriate estimates. *Cognitive Complexity/Depth of Knowledge Rating:* High
MA.912.T.5.2	Decide whether a solution is reasonable in the context of the original situation. *Remarks/Examples*: Example: Sandy was asked to solve for tan θ, given that the sin θ=3/5 where $\mathbf{0}^0 \leq \theta < \mathbf{360}^0$. When she completed her work, her answer was tan θ=3/4. Was her answer reasonable for the given problem? Justify your decision. *Cognitive Complexity/Depth of Knowledge Rating:* Moderate
MA.912.T.5.3	Determine whether a given trigonometric statement is always, sometimes, or never true. Use the properties of the real numbers, order of operations, and trigonometric identities to justify the steps involved in verifying identities and solving equations. *Remarks/Examples*:

	Example: Is the statement $sin^2x = 2sinxcosx$ true for all x? Explain your answer. Example: Verify that $\frac{sin2\alpha}{1-cos2\alpha} = cot2\alpha$ by justifying each step. *Cognitive Complexity/Depth of Knowledge Rating:* High